

Forward Modelling Galaxy Surveys:

An Analytical Model of the Non-Locality and Stochasticity of the Galaxy-Halo Connection in Spherical Projection

Maximilian von Wietersheim-Kramsta
Cosmic Rhythms, CBPF, Rio de Janeiro

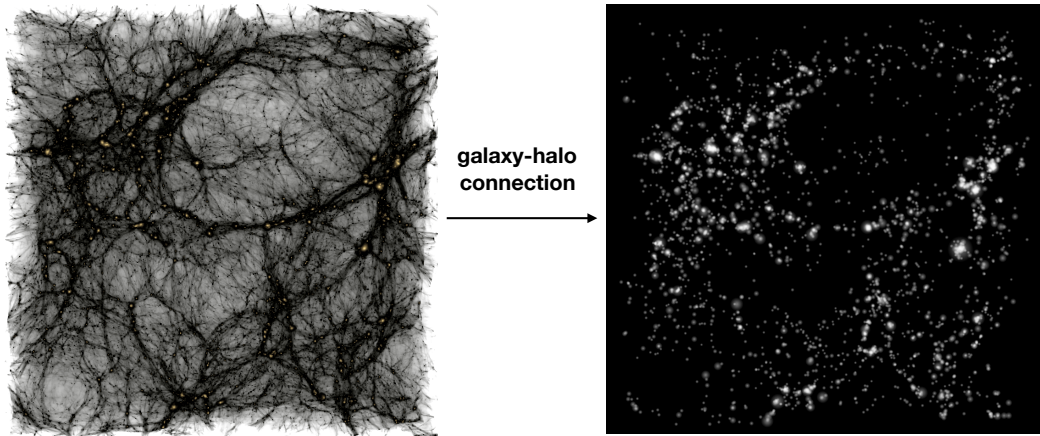
 [mwiet.github.io](https://github.com/mwiet)

9th of September 2026

In collaboration with Nicolas Tessore, Benjamin Joachimi, Qianjun Hang, Niall Jeffrey, Joop Schaye

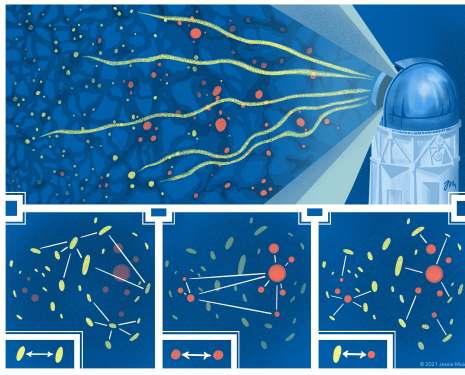


Galaxy Bias: Galaxies as Biased Tracers of the Matter Distribution



Wechsler & Tinker (2018)

LSS Probes: Galaxy clustering, weak lensing, and galaxy-galaxy lensing



Credit: Jessie Muir

Weak lensing:

- 2pt: Source clustering enters at 2nd order
- >2pt: Source clustering contributes directly

Galaxy clustering:

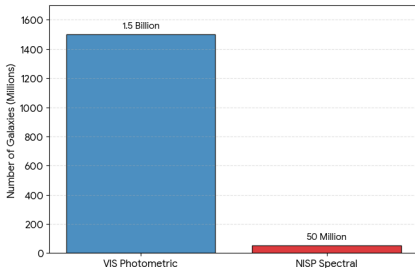
- Galaxy bias enters directly

Galaxy-galaxy lensing:

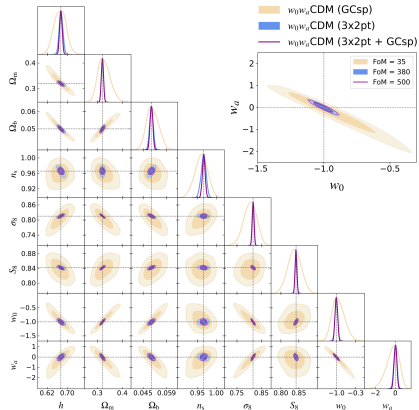
- Galaxy bias enters directly

Photometric galaxy surveys drive precision cosmology

Euclid: Photometry vs.
Spectroscopy



Forecast for Euclid DR3
cosmological constraints

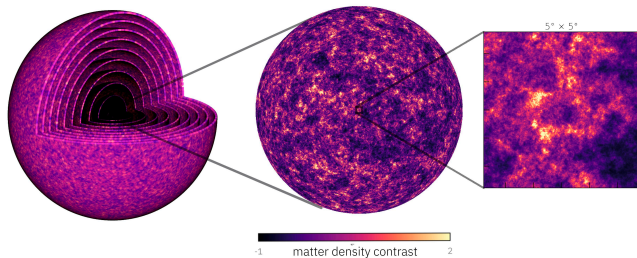


Mellier et al. (2025)

Photometric uncertainties & sky projection make the natural geometry spherical

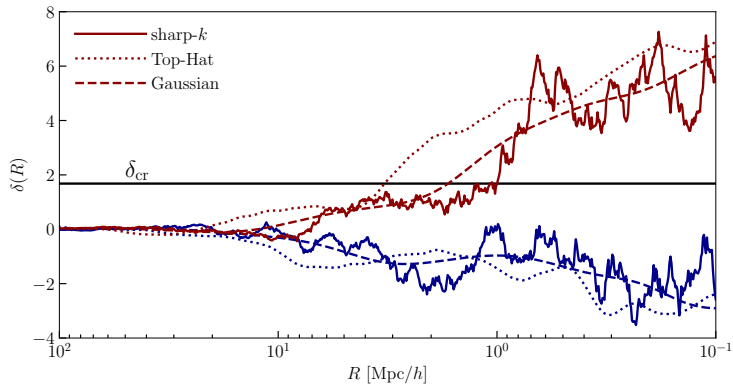
Projecting the 3D matter density field $\delta(x)$ onto the 2D celestial sphere, weighted by a redshift distribution $W(z)$:

$$\delta(\theta) = \sum_{\ell m} \delta_{\ell m 0} Y_{\ell m}(\theta) = \int dz W(z) \delta(\theta, z); \quad \delta(\theta, z) = \delta(x)$$



Tessore et al. (2023)

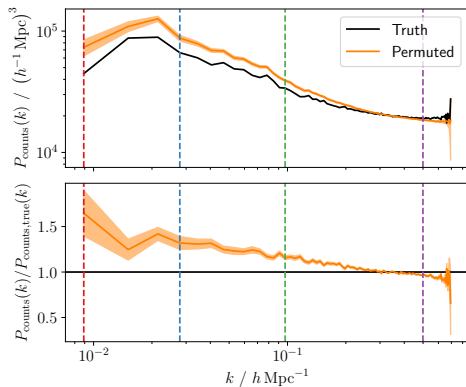
Beyond Determinism: The galaxy bias is stochastic



Desjacques, Jeong & Schmidt (2019)

Beyond LIMD: The galaxy bias is non-local

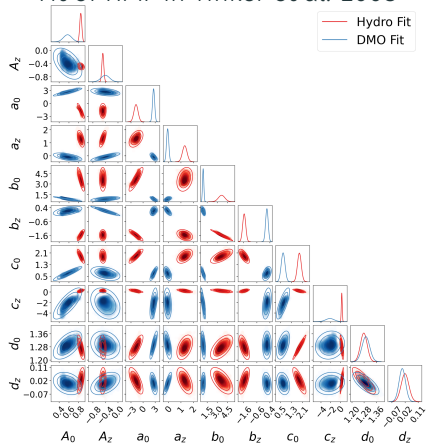
Power spectrum of the halo counts field before and after permuting voxels with approximately equal density



e.g. Bartlett, Ho & Wandelt (2024) See also Chan, Scoccimarro & Sheth (2012)

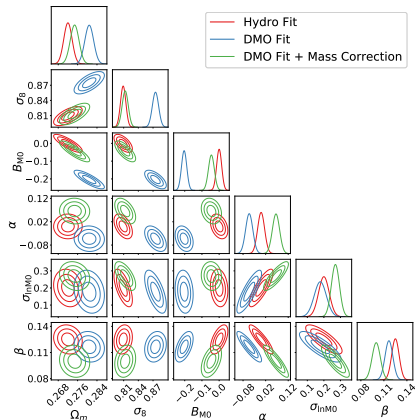
Baryon feedback & galaxy/halo bias are coupled

Fit of HMF in Tinker et al. 2008



Castro et al. (2026)

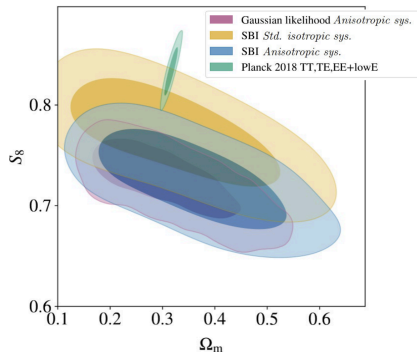
Λ CDM constraints with mock
Euclid-like cluster counts (6 bins)



Castro et al. (2026)

Simulation-based inference methods allow to incorporate all of this complexity

Weak lensing SBI with KiDS-1000
incl. 2pt source clustering

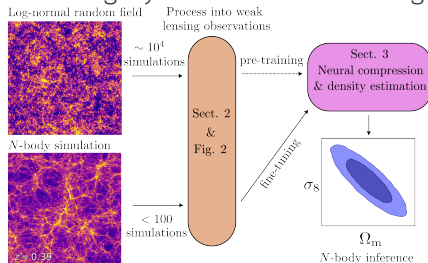


von Wietersheim-Kramsta, Lin et al. 2025
Lin, von Wietersheim-Kramsta et al. 2023

KiDS-Legacy 2pt SBI:

Lin, von Wietersheim-Kramsta et al. in prep.

Field-level multi-fidelity SBI with
KiDS-Legacy incl. source clustering



Saoulis, ..., von Wietersheim-Kramsta et al.
2026

KiDS-Legacy field-level SBI on data:

Saoulis, ..., von Wietersheim-Kramsta et al. in
prep.

Galaxy Bias Model Wishlist

- Make use of the photometric galaxies $\Rightarrow$ **Spherical projection** $\Rightarrow$ Halo mass, scale radius, or substructure hierarchy are not available...

Galaxy Bias Model Wishlist

- Make use of the photometric galaxies $\Rightarrow$ **Spherical projection** $\Rightarrow$ Halo mass, scale radius, or substructure hierarchy are not available...
- Take into account additional effects in the galaxy-halo connection at linear and quasi-linear scales
 - $\Rightarrow$ **Stochasticity** (baryon feedback, initial conditions, etc.)
 - $\Rightarrow$ **Non-locality** (tidal effects, environmental dependence, etc.)

Galaxy Bias Model Wishlist

- Make use of the photometric galaxies $\Rightarrow$ **Spherical projection** $\Rightarrow$ Halo mass, scale radius, or substructure hierarchy are not available...
- Take into account additional effects in the galaxy-halo connection at linear and quasi-linear scales
 - $\Rightarrow$ **Stochasticity** (baryon feedback, initial conditions, etc.)
 - $\Rightarrow$ **Non-locality** (tidal effects, environmental dependence, etc.)
- Flexible enough to be informed by hydrodynamical simulations to incorporate **baryon physics**

Galaxy Bias Model Wishlist

- Make use of the photometric galaxies $\Rightarrow$ **Spherical projection** $\Rightarrow$ Halo mass, scale radius, or substructure hierarchy are not available...
- Take into account additional effects in the galaxy-halo connection at linear and quasi-linear scales
 - $\Rightarrow$ **Stochasticity** (baryon feedback, initial conditions, etc.)
 - $\Rightarrow$ **Non-locality** (tidal effects, environmental dependence, etc.)
- Flexible enough to be informed by hydrodynamical simulations to incorporate **baryon physics**
- Be **computationally efficient** for inference (both for likelihood-based sampling and simulation-based inference)

BOLTS Model: Bias of Overdensities in Large-Scale Tracers on the Sphere

Linear bias model in spherical harmonic space:

$$\delta_{\ell m}^b = b_{\ell m} \delta_{\ell m}. \quad (1)$$

Enforcing homogeneity of each field:

$$\begin{aligned} \langle \delta_{\ell m} (\delta_{\ell' m'}^b)^* \rangle &= \langle \delta_{\ell m}^b \delta_{\ell' m'}^* \rangle = \delta_{\ell \ell'} \delta_{m m'} C_{\ell}^b \\ &= \delta_{\ell \ell'} \delta_{m m'} \langle b_{\ell} \rangle C_{\ell}, \end{aligned} \quad (2)$$

$\Rightarrow |b_{\ell m}|^2$ must be independent of m , i.e. $\langle |b_{\ell m}| \rangle = \langle |b_{\ell}| \rangle \forall m$.

BOLTS Model: Bias of Overdensities in Large-Scale Tracers on the Sphere

As $\langle |b_{\ell m}| \rangle = \langle |b_\ell| \rangle \forall m$ is only true in expectation, we may add a random contribution to the bias model:

$$b_{\ell m} = \beta_\ell + \gamma_{\ell m}, \quad (3)$$

where $\gamma_{\ell m} = \epsilon_{\ell m} + i\eta_{\ell m} \in \mathbb{C}$ with $\epsilon_{\ell m}, \eta_{\ell m} \in \mathbb{R}$.

$\langle \gamma_{\ell m} \rangle_m = 0 \forall \ell$ to maintain homogeneity.

BOLTS Model: Bias of Overdensities in Large-Scale Tracers on the Sphere

Simplest model with the allowed degrees of freedom:

$$b_{\ell m} \sim \mathcal{CN}(\beta_\ell + i0, \mathbf{\Sigma}_\ell), \quad (4)$$

where $\mathcal{CN}$ is a complex Gaussian or normal distribution.

In general,

$$\begin{pmatrix} \epsilon_{\ell m} & \eta_{\ell m} \end{pmatrix} \sim \mathcal{N}\left(\boldsymbol{\mu}_\ell = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{\Sigma}_\ell = \begin{pmatrix} \sigma_{1,\ell}^2 & \rho \sigma_{1,\ell} \sigma_{2,\ell} \\ \rho \sigma_{1,\ell} \sigma_{2,\ell} & \sigma_{2,\ell}^2 \end{pmatrix}\right). \quad (5)$$

In its simplest form, $\rho = 0$.

BOLTS Model: Bias of Overdensities in Large-Scale Tracers on the Sphere

Complete model for redshift bin j :

$$b_{\ell m}^{(j)} \sim \mathcal{CN} \left(\beta_{\ell}^{(j)} + i 0, \mathbf{\Sigma}_{\ell}^{(j)} = \begin{pmatrix} [\sigma_{1,\ell}^{(j)}]^2 & 0 \\ 0 & [\sigma_{2,\ell}^{(j)}]^2 \end{pmatrix} \right). \quad (6)$$

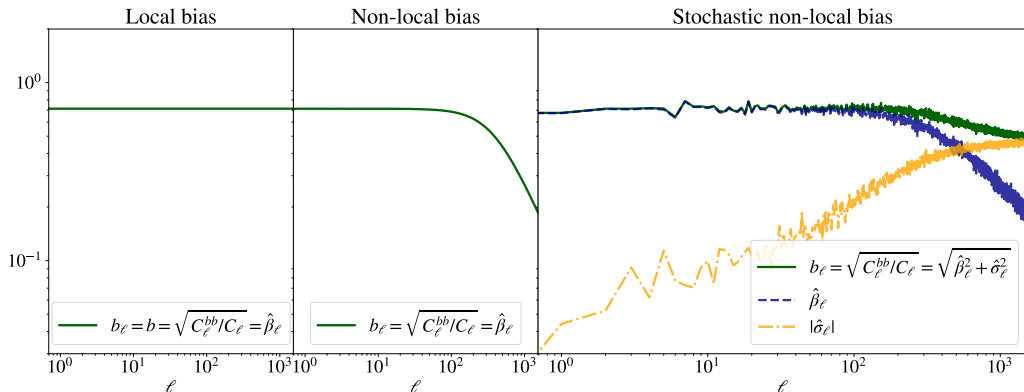
With a predicted angular power spectrum of the biasing field:

$$\begin{aligned} \langle |b_{\ell m}|^2 \rangle &= \langle (\beta_{\ell} + \epsilon_{\ell})^2 + \eta_{\ell}^2 \rangle \\ &= \beta_{\ell}^2 + \langle \epsilon_{\ell}^2 \rangle + \langle \eta_{\ell}^2 \rangle \\ &= \beta_{\ell}^2 + \sigma_{1,\ell}^2 + \sigma_{2,\ell}^2. \end{aligned} \quad (7)$$

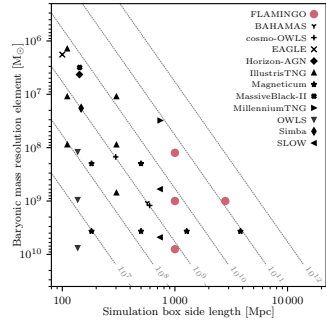
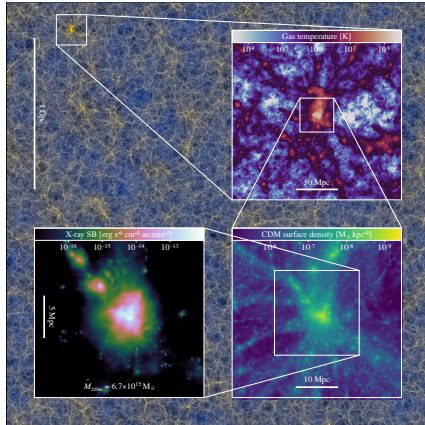
BOLTS Model: Bias of Overdensities in Large-Scale Tracers on the Sphere

As opposed to deterministic local bias linear bias:

- β_ℓ captures the scale-dependent bias
- $\sigma_{1,\ell}^2 + \sigma_{2,\ell}^2$ captures the scale-dependent stochasticity and non-locality



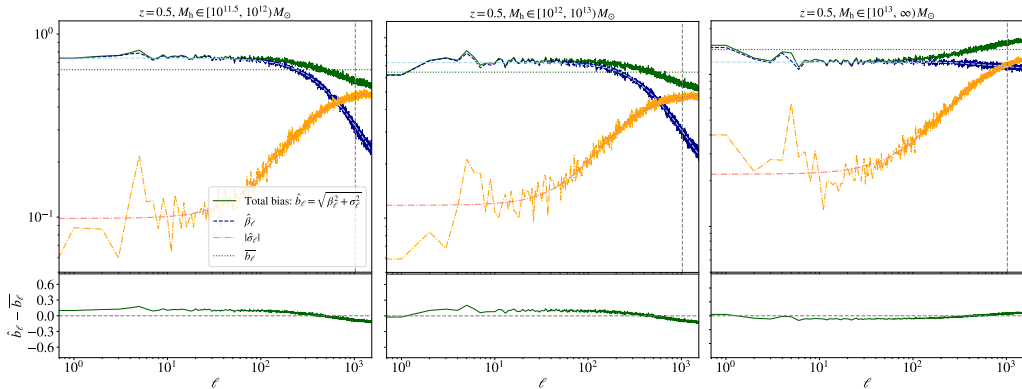
Testing with Hydro Simulations: FLAMINGO



Schaye et al. (2023)

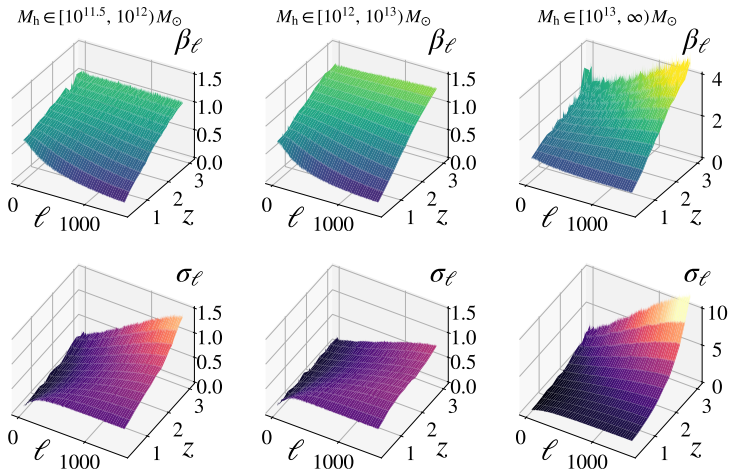
BOLTS Model: Non-locality and stochasticity naturally emerges in hydro sims

As measured from FLAMINGO lightcones for different halo mass bins:

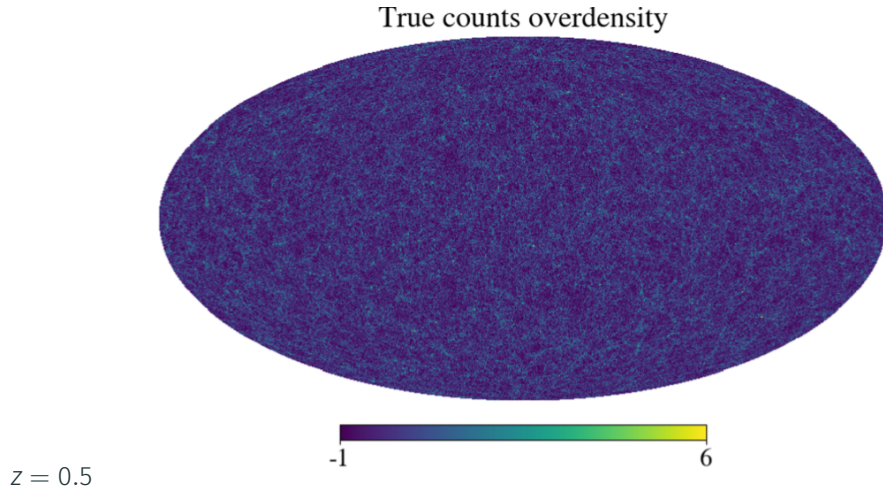


BOLTS Model: Non-locality and stochasticity naturally emerges in hydro sims

As measured from FLAMINGO lightcones:

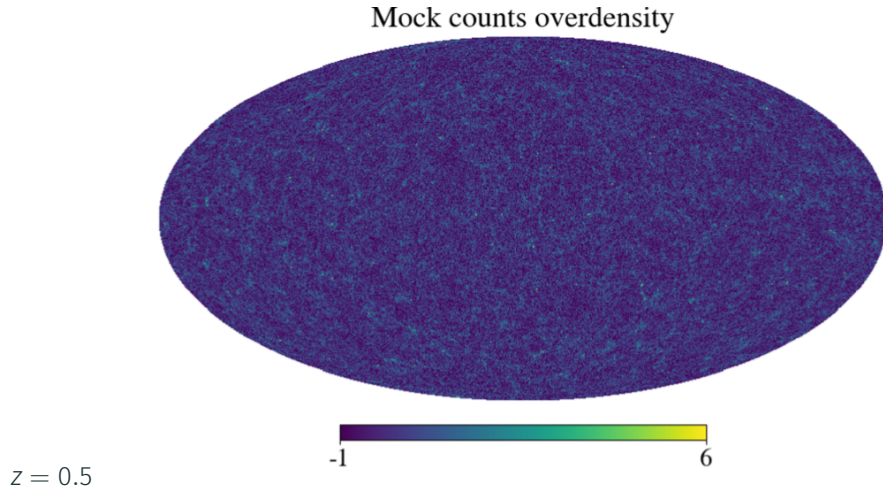


BOLTS Model: Bias of Overdensities in Large-Scale Tracers on the Sphere



von Wietersheim-Kramsta et al. (in prep.)

BOLTS Model: Bias of Overdensities in Large-Scale Tracers on the Sphere

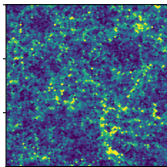


von Wietersheim-Kramsta et al. (in prep.)

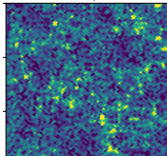
BOLTS model reproduces the galaxy field better than local deterministic bias

$z=0.5, M_h \in [10^{11.5}, 10^{12}) M_\odot$

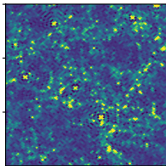
True



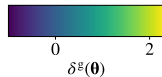
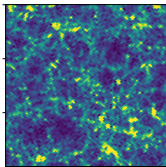
BOLTS mock (1 realisation)



Local mock from δ^c



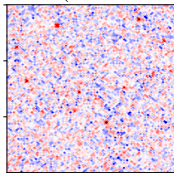
Local mock from δ^m



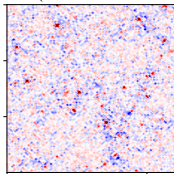
BOLTS model reproduces the galaxy field better than local deterministic bias

$z = 0.5, M_h \in [10^{11.5}, 10^{12}) M_\odot$

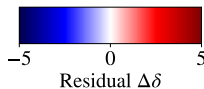
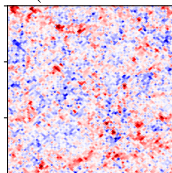
Residual (BOLTS - True)



Residual (Local from δ^c - True)



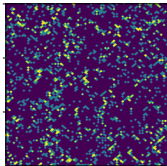
Residual (Local from δ^m - True)



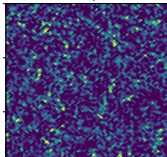
BOLTS model reproduces the galaxy field better than local deterministic bias

$z = 1.5, M_h \in [10^{13}, \infty) M_\odot$

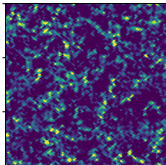
True



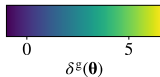
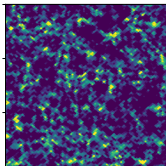
BOLTS mock (1 realisation)



Local mock from δ^c



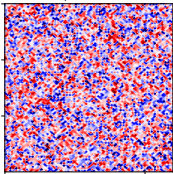
Local mock from δ^m



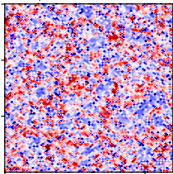
BOLTS model reproduces the galaxy field better than local deterministic bias

$z = 1.5, M_h \in [10^{13}, \infty) M_\odot$

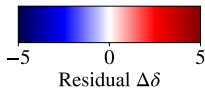
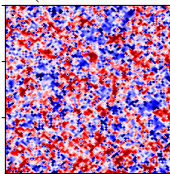
Residual (BOLTS - True)



Residual (Local from δ^c - True)

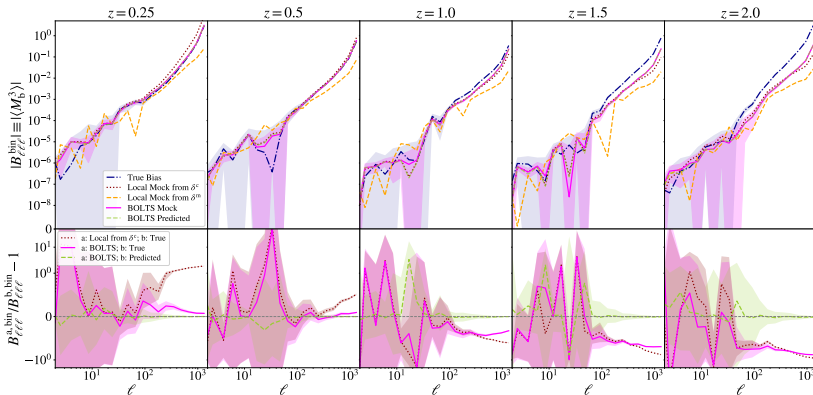


Residual (Local from δ^m - True)



BOLTS model reproduces the bispectrum better than local deterministic bias

Equilateral term of angular bispectrum for $M_h \in [10^{13}, \infty)$



$10^{11.5} M_\odot < M_h < 10^{13} M_\odot$: accurate down to 11-27 Mpc over $0.25 \leq z \leq 1.5$ and to 9-19 Mpc at $2 \leq z \leq 3$

$M_h > 10^{13} M_\odot$: accurate down to 3-4.5 Mpc over $0.25 \leq z \leq 0.5$, to 27-35 Mpc at $1 \leq z \leq 1.5$, and to 50-90 Mpc at $2 \leq z \leq 3$

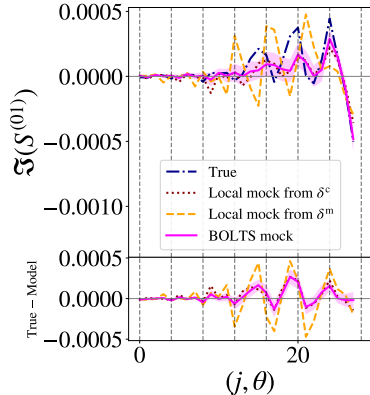
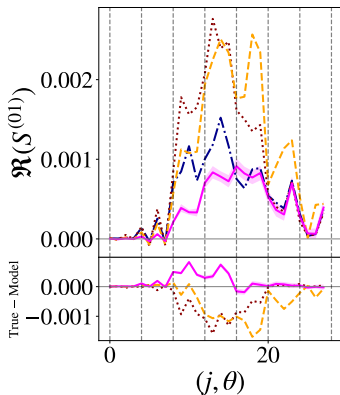
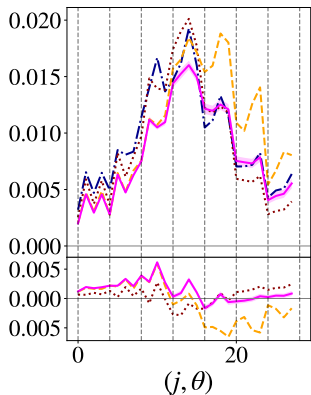
Covariance between phase-harmonic wavelet coefficients:

$$C_{\lambda,p,\lambda',p'}(\tau) = \text{Cov}\left([X * \psi_{\lambda}(\mathbf{r})]^p, [X * \psi_{\lambda'}(\mathbf{r} + \tau)]^{p'}\right), \quad [z]^p = |z| e^{ip\varphi} \quad (8)$$

- S_{11} ($p=1, \lambda=\lambda'$): wavelet power spectrum, equivalent to 2-point statistics; fully characterises Gaussian fields
- S_{01} ($p=0, \lambda=\lambda'$): amplitude-phase coupling, vanishes for Gaussian fields; signals non-linear structure
- C_{01} ($p=0, \lambda \neq \lambda'$): cross-scale mode coupling, sensitive to coherent structures (filaments, sheets)

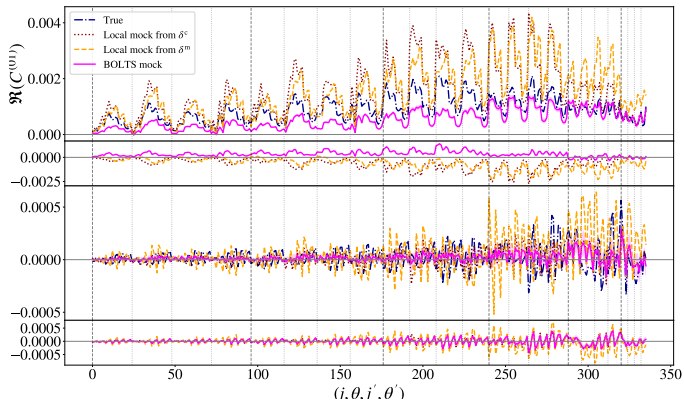
BOLTS Model: WPH, $M_h \in [10^{11.5}, 10^{12}) M_\odot$, $z \in [0.50, 0.55)$

$z=0.5$, $M_h \in [10^{11.5}, 10^{12}) M_\odot$



S_{11} : wavelet power spectrum, S_{01} : amplitude-phase coupling

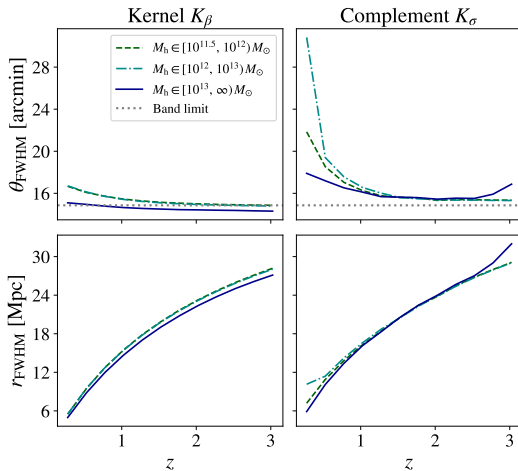
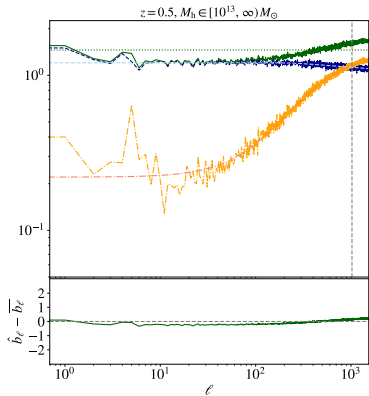
BOLTS Model: WPH, $M_h \in [10^{11.5}, 10^{12}) M_\odot$, $z \in [0.25, 0.30)$



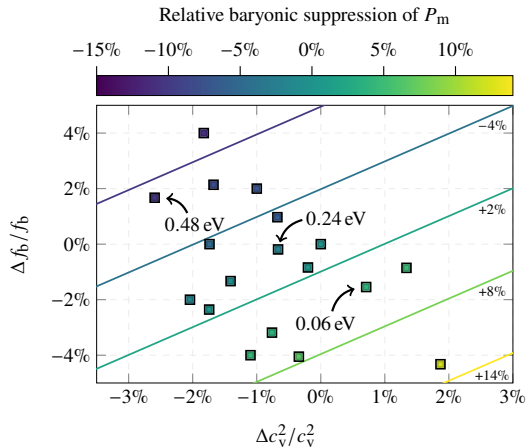
C_{01} : cross-scale mode coupling, sensitive to coherent structures

von Wietersheim-Kramsta et al. (in prep.)

Non-Locality & Stochasticity of Galaxy Bias Matters on Interesting Scales...

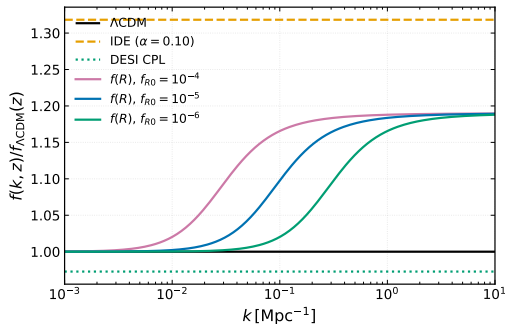


At these scales, baryons, neutrinos, and modified gravity impact LSS



Elbers et al. (2024)

Scale- and redshift-dependent growth $f\sigma_8(z; k)$ in Hu-Sawicki $f(R)$



Randrianjanahary et al. (2026)

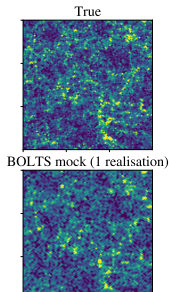
Conclusions & Outlook

BOLTS: minimal harmonic-space bias, **inherently non-local & stochastic**. One the **arXiv** soon...

- β_ℓ (deterministic) & σ_ℓ (stochastic), closed-form estimators from **2-point calibration** only
- Reproduces **bispectrum** & **WPH moments** beyond local bias
- $\beta_\ell(z)$ & $\sigma_\ell(z) \Rightarrow$ fast **forward model**/SBI on *Euclid*, *Rubin*, *Roman*

Outlook:

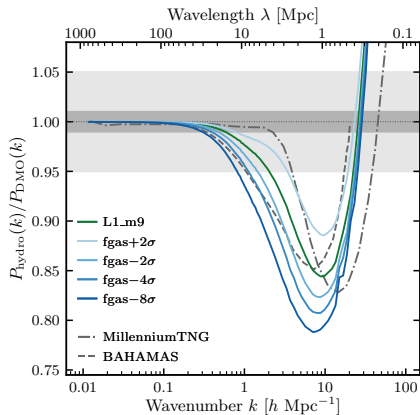
- **Baryon feedback model** & cosmology dependence of β_ℓ, σ_ℓ across FLAMINGO variants
- **Modified gravity** & **neutrino mass** imprints on scale-dependent bias
- Assembly bias: condition on stellar mass, colour, SFR



Questions?

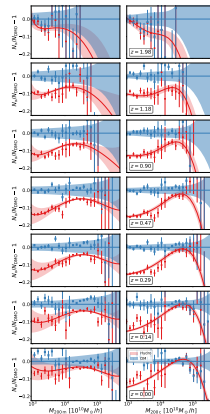
Baryon Feedback

Relative power spectrum
suppression



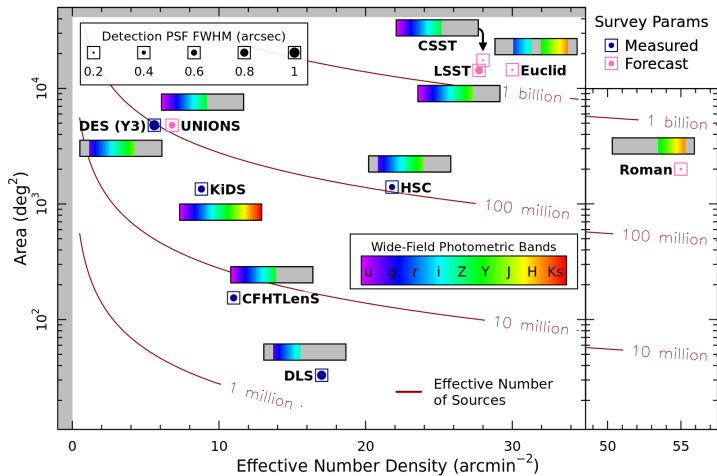
Schaye et al. (2023)

HMF hydro (red)
vs. DMO (blue)



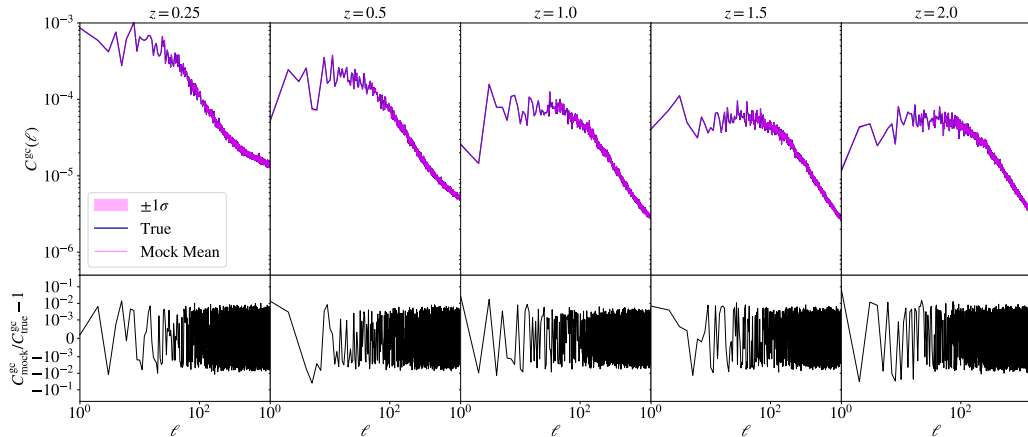
Castro et al. (2026)

Motivation: Unprecedented Precision



BOLTS Model: Bias of Overdensities in Large-Scale Tracers on the Sphere

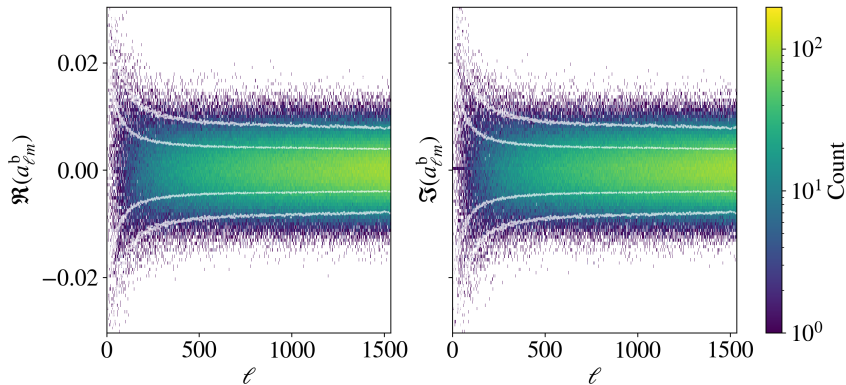
FLAMINGO vs. BOLTS



BOLTS Model: Bias of Overdensities in Large-Scale Tracers on the Sphere

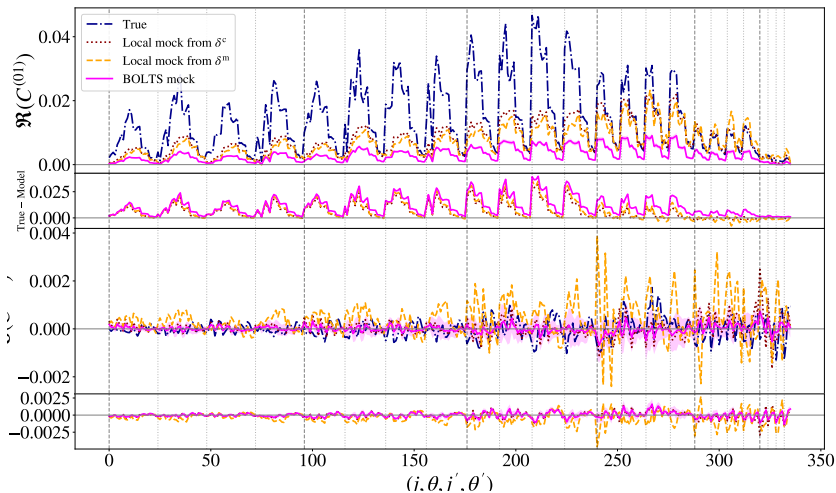
As measured from FLAMINGO lightcones:

$$z=0.25, M_h \in [10^{13}, \infty) M_\odot$$



Consistent with Gaussian distribution of harmonic coefficients, as assumed in the BOLTS model.

BOLTS Model: WPH, $M_h > 10^{13} M_\odot$, $z \in [1.50, 1.55)$



C_{01} : cross-scale mode coupling