Subhalo Detection with Simulation-Based Inference from Galaxy-Scale Strong Lenses

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Institute for Computational Cosmology & Centre for Extragalactic Astronomy, Durham University

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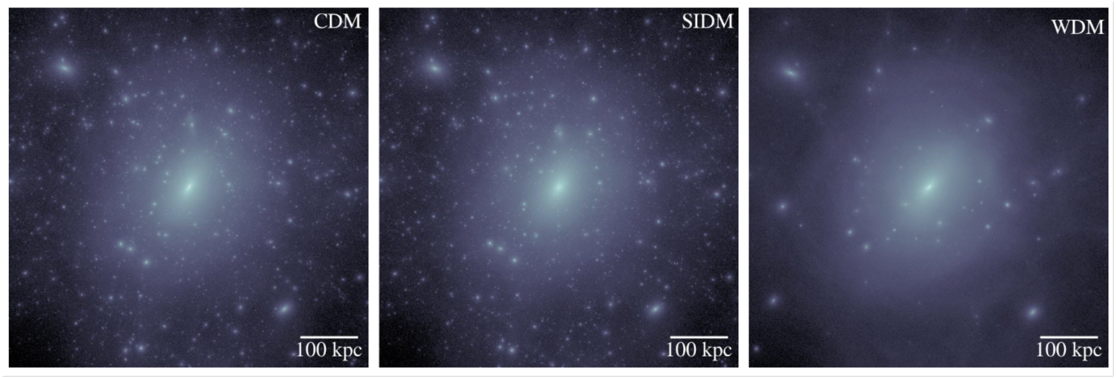
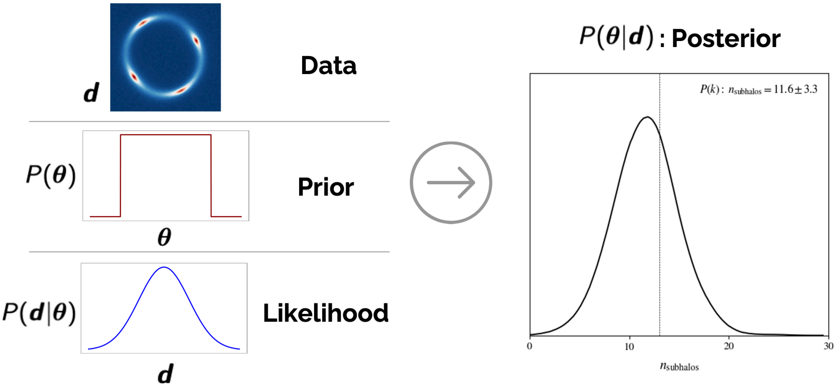
The Nature fo Dark Matter: Substructure Detection



Bullock and Boylan-Kolchin (2017)

Bayesian Inference

$$P(\boldsymbol{\theta} \mid \boldsymbol{d}) = \frac{P(\boldsymbol{d} \mid \boldsymbol{\theta})\, P(\boldsymbol{\theta})}{P(\boldsymbol{d})} \tag{1}$$

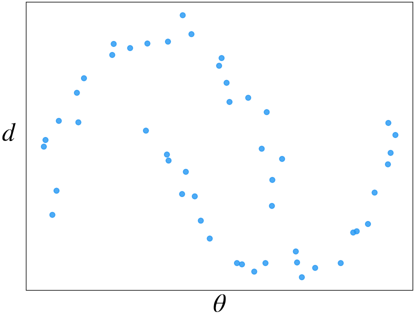


Bayesian Inference: The Joint Probability

$$P(\boldsymbol{\theta} \mid \boldsymbol{d}) = \frac{P(\boldsymbol{d} \mid \boldsymbol{\theta})\, P(\boldsymbol{\theta})}{P(\boldsymbol{d})} \propto P(\boldsymbol{\theta}, \boldsymbol{d})\, P(\boldsymbol{\theta}) \tag{2}$$

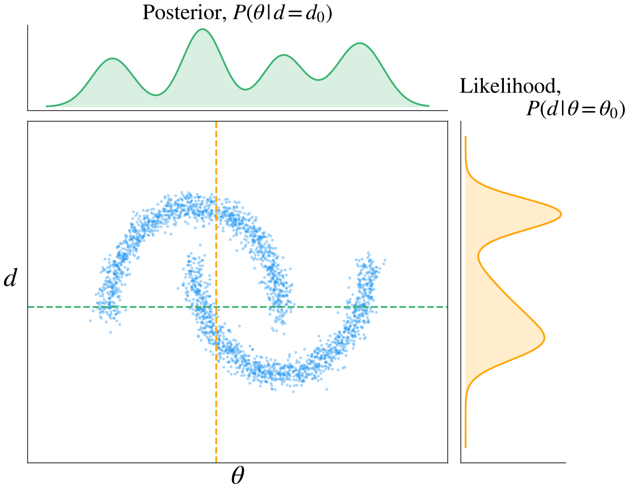
Joint probability: $P(\boldsymbol{\theta}, \boldsymbol{d} \mid \mathrm{Model})$

Simulator: $\boldsymbol{d}_i \sim P(\boldsymbol{d} \mid \boldsymbol{\theta}, \mathrm{Model})$



Bayesian Inference: The Joint Probability

Joint probability: $P(\boldsymbol{\theta}, \boldsymbol{d} \mid \mathrm{Model})$

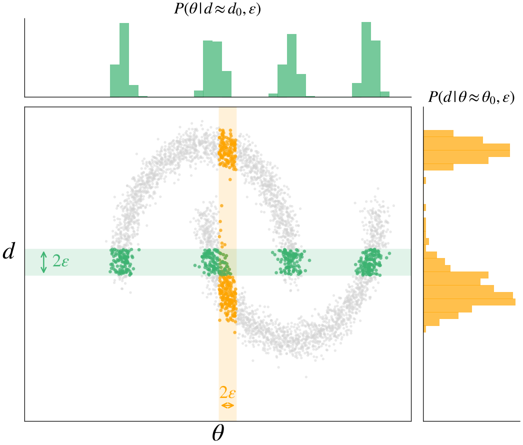


Simulation-Based Inference

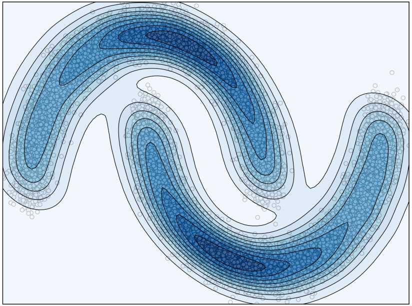
Simplest Case: Approximate Bayesian Computation

Converges given:

$$\lim_{\epsilon \to 0} P_{\mathrm{ABC}}(\theta \mid d_0) = P(\theta \mid d_0). \tag{3}$$



Neural Posterior Estimation (NPE)



$D_{KL}(P \,||\, Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)}$

See Papamakarios and Murray (2016); Lueckmann et al. (2017); Greenberg et al. (2019); Cranmer et al. (2020)

1. Draw simulations:

$$\boldsymbol{d}^* \sim P(\boldsymbol{d} \mid \boldsymbol{\theta}^*); \;\; \boldsymbol{\theta}^* \sim P(\boldsymbol{\theta}). \tag{4}$$

2. Find an estimator of the posterior, $\hat{P}_w(\boldsymbol{\theta} \mid \boldsymbol{d})$, with its weights, $\boldsymbol{w}$, such that:

$$\boldsymbol{w}^* = \arg\min_{w} \mathbb{E}_{P(\boldsymbol{d})}[\, D_{\mathrm{KL}}(P(\boldsymbol{\theta} \mid \boldsymbol{d}) \,||\, \hat{P}_w(\boldsymbol{\theta} \mid \boldsymbol{d}))], \tag{5}$$

$$\boldsymbol{w}^* = \arg\max_{w} \mathbb{E}_{P(\boldsymbol{\theta},\boldsymbol{d})}[\ln(\hat{P}_w(\boldsymbol{\theta} \mid \boldsymbol{d}))]. \tag{6}$$

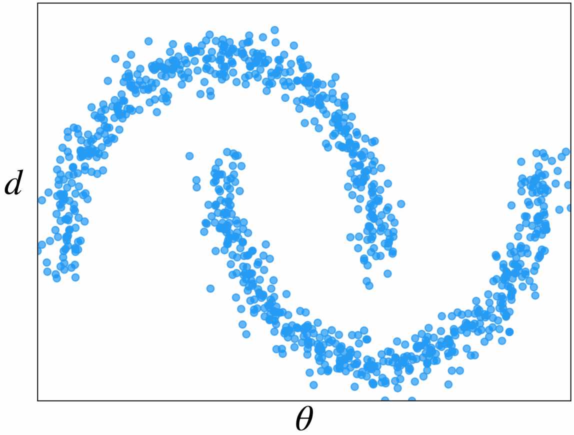
3. Train a neural network from this loss function:

$$L(\boldsymbol{w}) = -\mathbb{E}_{P(\boldsymbol{\theta},\boldsymbol{d})}[\ln(\hat{P}_w(\boldsymbol{\theta} \mid \boldsymbol{d}))] \tag{7}$$

4. Use network to directly sample $\hat{P}_w(\boldsymbol{\theta} \mid \boldsymbol{d})$.

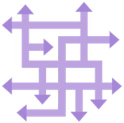
Neural Density Estimation: Normalising Flows

Learns **invertible and differentiable** transformations between any distribution and a Gaussian.



e.g. Masked Autoregressive Flows (MAFs)

Simulation-Based Inference



Signal and uncertainty modelling of arbitrary complexity (vary all complexities simultaneously)

$d_0, d_1, d_2 \ldots$

Amortisable (all model evaluations can be data-independent in NPE)

$t \rightarrow \Theta$

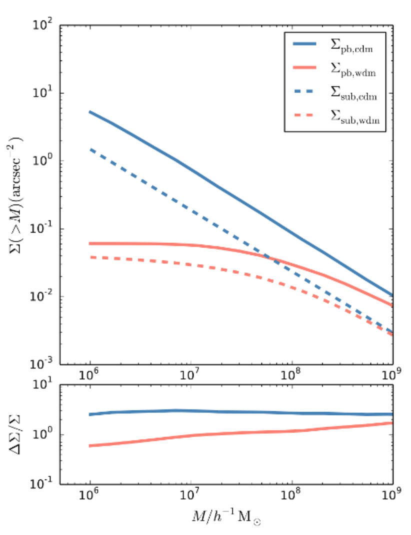
Bayesian uncertainty propagation from data to parameters

Likelihood can take an arbitrary form

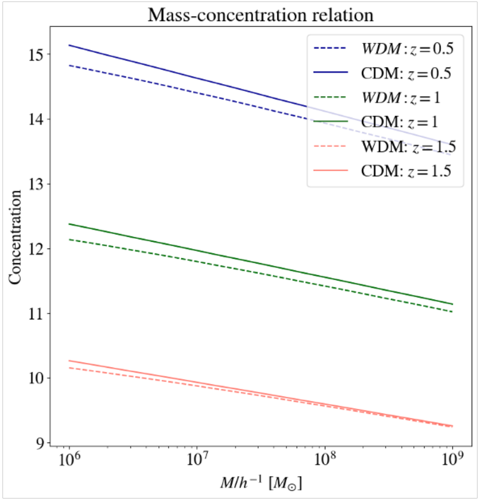
Subhalo Search:
Forward Modelling & Inference

Subhalo Search: Forward Modelling the Subhalo Field

Number densities of perturbing interlopers and subhalos



Li et al. (2017)



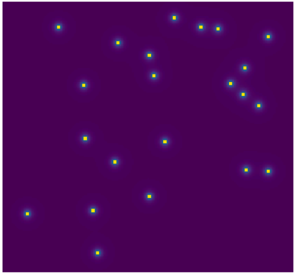
Ludlow et al. (2016)

Subhalo Search: Forward Modelling the Subhalo Field

Source:
- Elliptical Core-Sersic
- $z = 1$

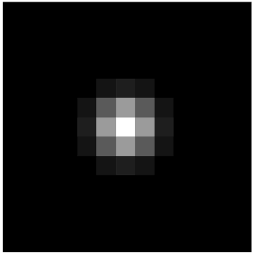
Lens:
- Power law mass
- $z = 0.5$
- No external shear

Perturbers:
- Warm Dark Matter
- Truncated NFW mass
- $M_{hf} = 10^7$
- $\mathbf{n_{subhalos}} \in \mathbf{[0, 30]}$



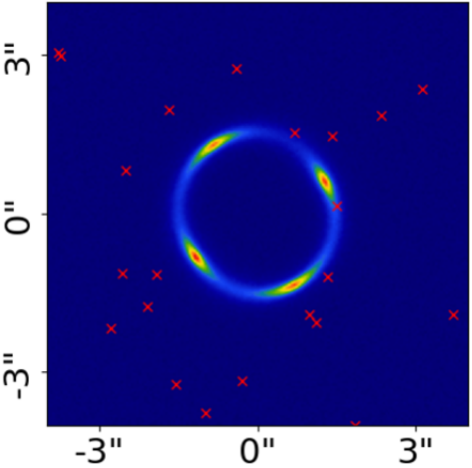
He et al. (2022)

Observational Effects (HST-like)
- Exposure = 8000s
- Sky background = 0.1
- Pixel scale = 0.05”
- σ_{PSF} = 0.05”
- + Poisson noise



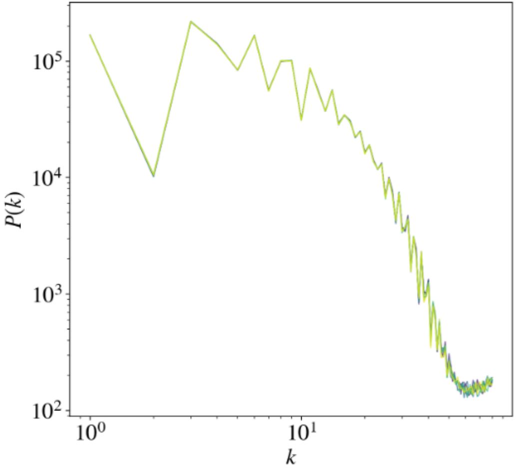
Subhalo Search: Forward Modelling the Subhalo Field

AutoLens: Mock Observations



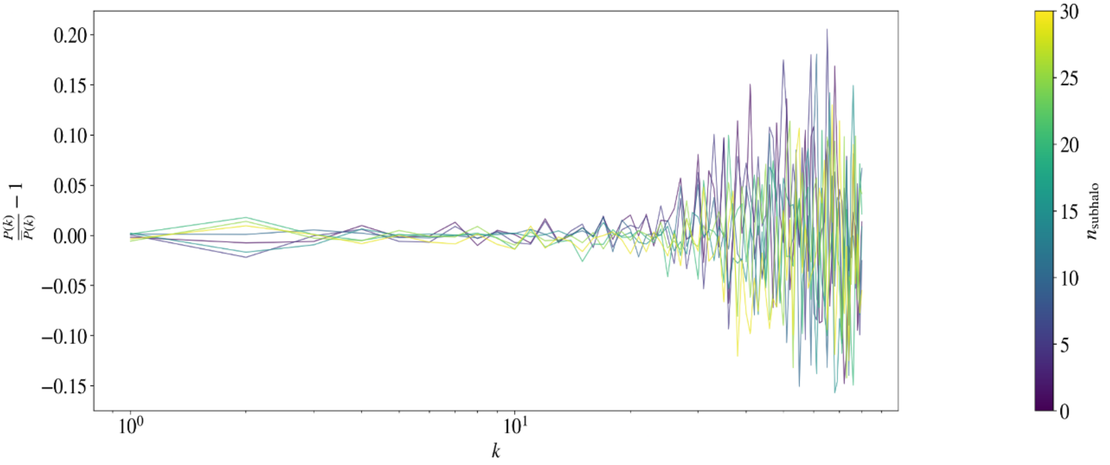
Nightingale et al. (2021)

Compression/summary statistic: $P(k)$



Repeat 1000 times...

Subhalo Search: Power Spectrum as a Summary



von Wietersheim-Kramsta et al. (in prep.)

Subhalo Search: Inference from the Power Spectrum

Ensemble of 2 NDEs

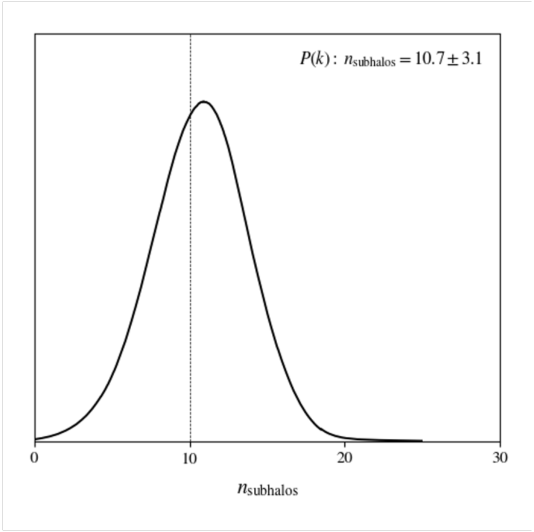
2 x MAFs

5 x MADEs

2 hidden layers

50 neurons each

Posterior



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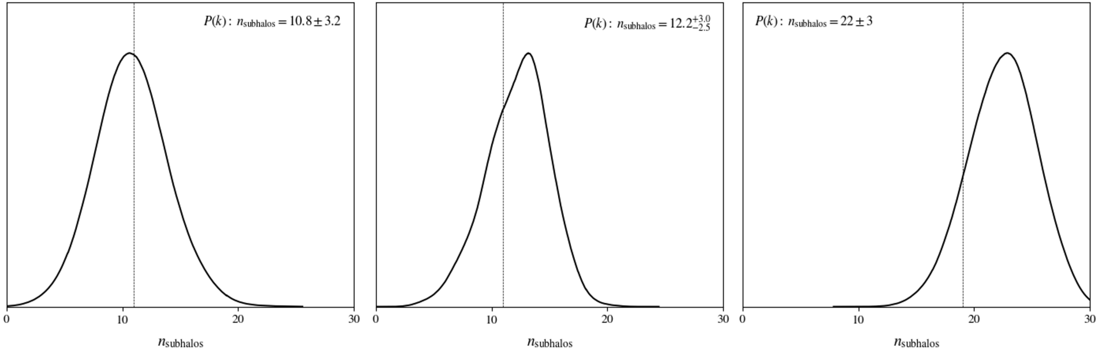

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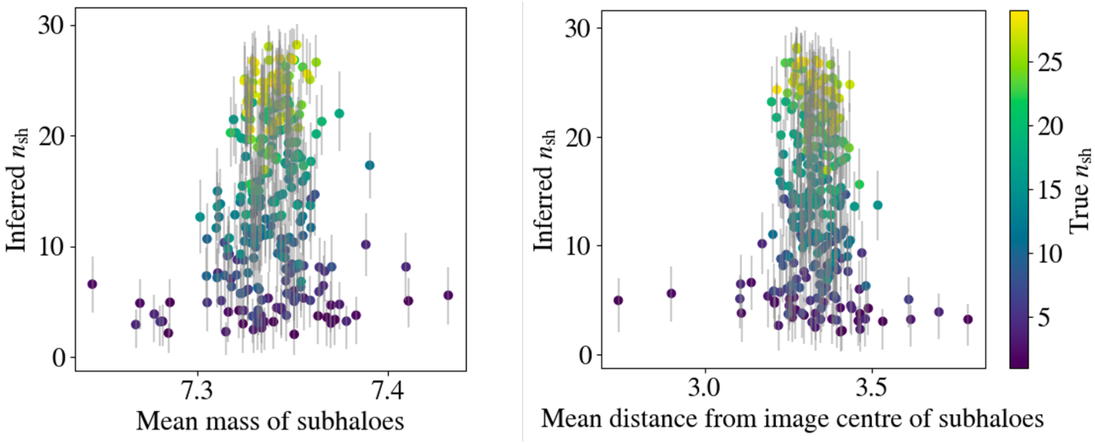
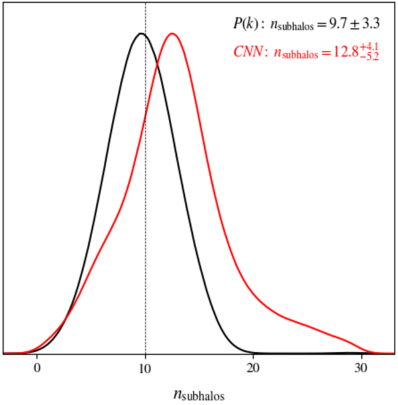


Subhalo Search: Inference with Other Summaries

Other compression schemes/summary statistics:

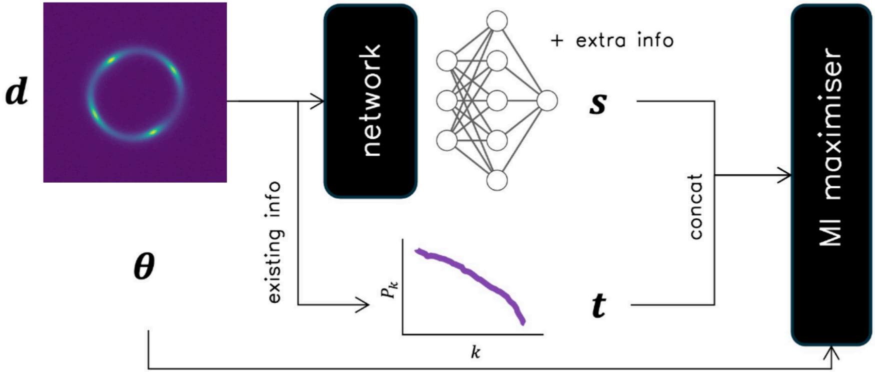
Convolutional Neural Networks (CNNs)



von Wietersheim-Kramsta et al. (in prep.)

CNNs can help recover other lens parameters, but lose information on the subhalo field

Subhalo Search: Hybrid Summaries



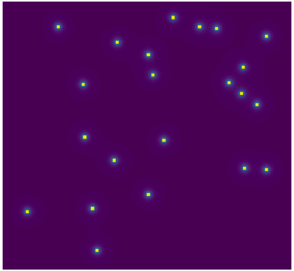
Makinen et al. (2025)

Subhalo Search: Forward Modelling the Subhalo Field & the Macro Model

Source:
- Elliptical Core-Sersic
- $z = 1$
- **Axis ratio $\in$ [0.3, 0.85)**
- **Axial tilt $\in$ [30, 70]°**

Lens:
- Power law mass
- $z = 0.5$
- No external shear
- $\mathbf{R_E \in [1.0, 1.5]''}$

Perturbers:
- Warm Dark Matter
- Truncated NFW mass
- $M_{hf} = 10^7$
- $\mathbf{n_{subhalos} \in [0, 30]}$



He et al. (2022)

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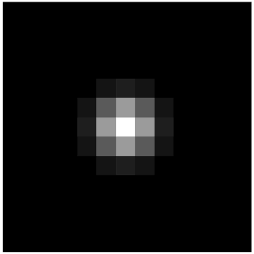
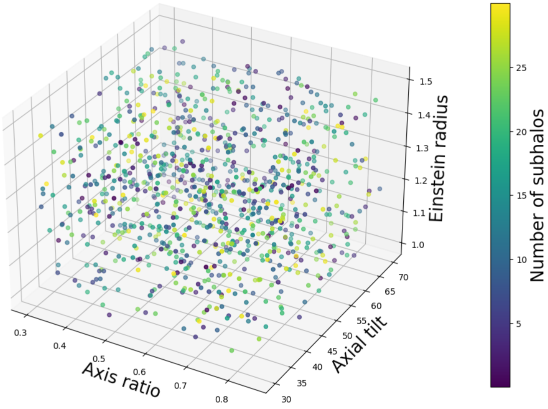


Forward Modelling the Subhalo Field & the Macro Model

Latin Hypercube:



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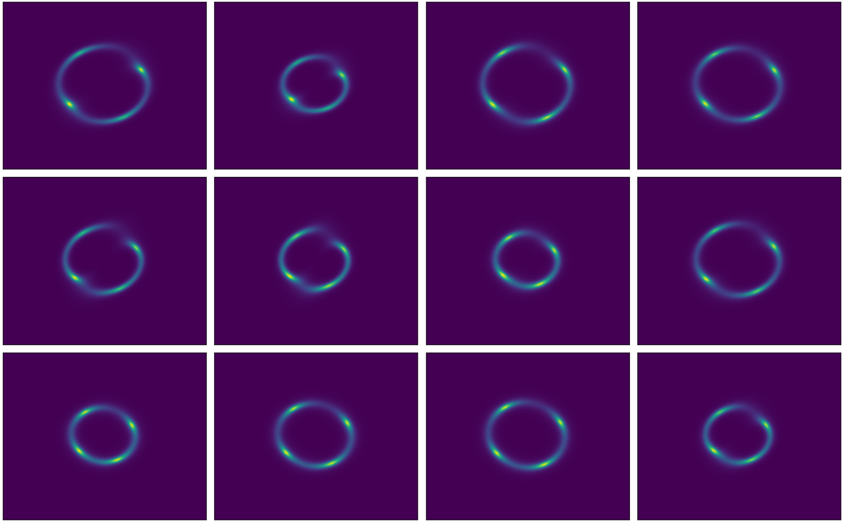
Forward Modelling the Subhalo Field & the Macro Model



von Wietersheim-Kramsta et al. (in prep.)

Conclusion & Outlooks

Conclusion & Outlooks

- SBI is a powerful method to incorporate model complexity & systematics
- We **accurately and robustly** recover the subhalo field from mocks
- The P(k) encodes most of the information on the subhalos
- SBI allows for **simultaneous** varying of subhalo & macro model parameters

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- **Future outlooks**:
  - Scale up to higher-dimensional parameter space
  - Incorporate dark matter models (**SIDM!**)
  - Add realistic systematics
  - Apply to data

Questions?

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