Simulation-Based Inference of Dark Matter Subhaloes

Forward modelling galaxy-scale strong lenses

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Durham Centre for Extragalactic Astronomy

Durham University

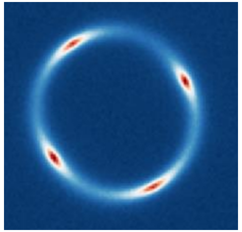
UKRI Science and Technology Facilities Council

Dark Matter under the Gravitational Lens, Hong Kong

In collaboration with: Richard Massey, Qiuhan He, James Nightingale, Andrew Robertson, Aristeidis Amvrosiadis, Leo Fung, Samuel Lange, Carlos Frenk, Shaun Cole, Ran Li, et al.

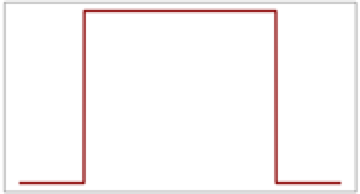
Recipe for Cosmological Inference

$\boldsymbol{d}$



Data

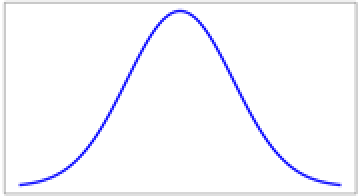
$P(\theta)$



$\boldsymbol{\theta}$

Prior

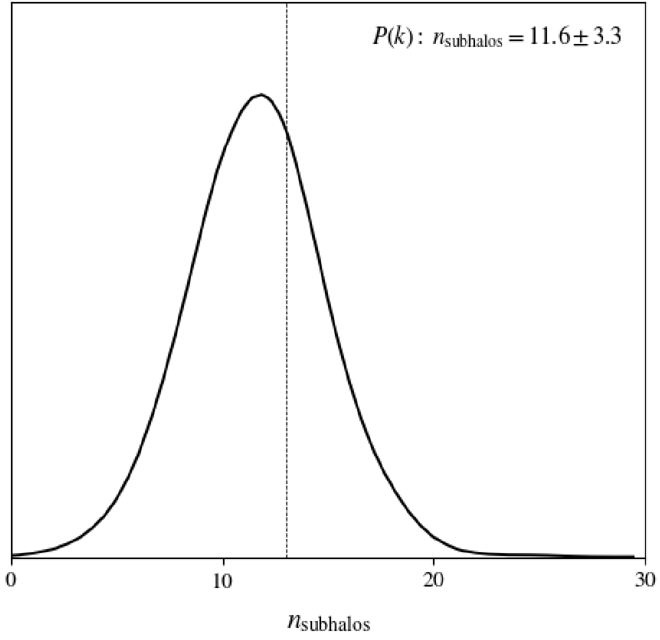
$P(\boldsymbol{d}|\boldsymbol{\theta})$



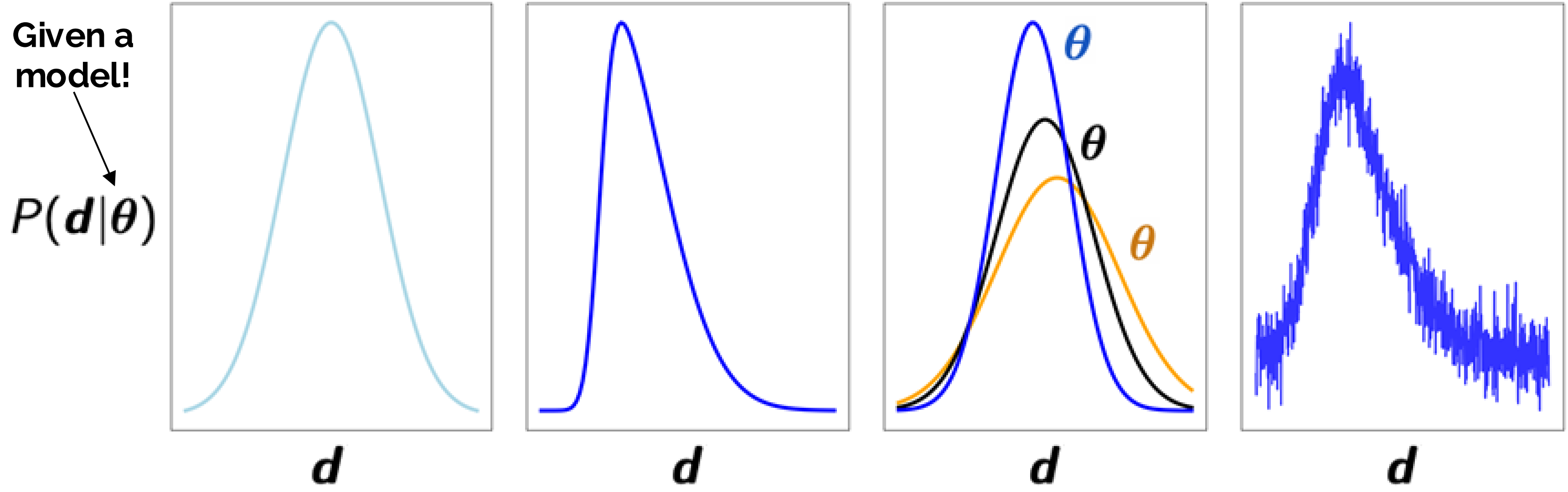
$\boldsymbol{d}$

Likelihood

$P(\boldsymbol{\theta}|\boldsymbol{d})$: **Posterior**



Modelling Likelihoods



Analytic

e.g.

$P(d|\theta) \propto e^{-(d-\mu)^2}$

Biased

e.g.

Instrumental systematics

Signal-dependent uncertainty

e.g.

Cosmic variance

Intractable

e.g.

Pixel-level effects

Bayes' Theorem

Posterior

Likelihood

Prior

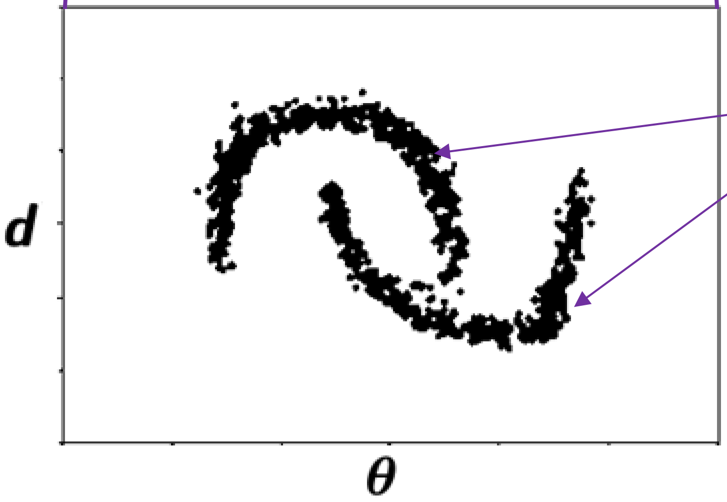
Joint probability

$$P(\boldsymbol{\theta}|\boldsymbol{d}) = \frac{P(\boldsymbol{d}|\boldsymbol{\theta}) \cdot P(\boldsymbol{\theta})}{P(\boldsymbol{d})} \propto P(\boldsymbol{\theta}, \boldsymbol{d}) \cdot P(\boldsymbol{\theta})$$

$\boldsymbol{\theta}$: Model parameters

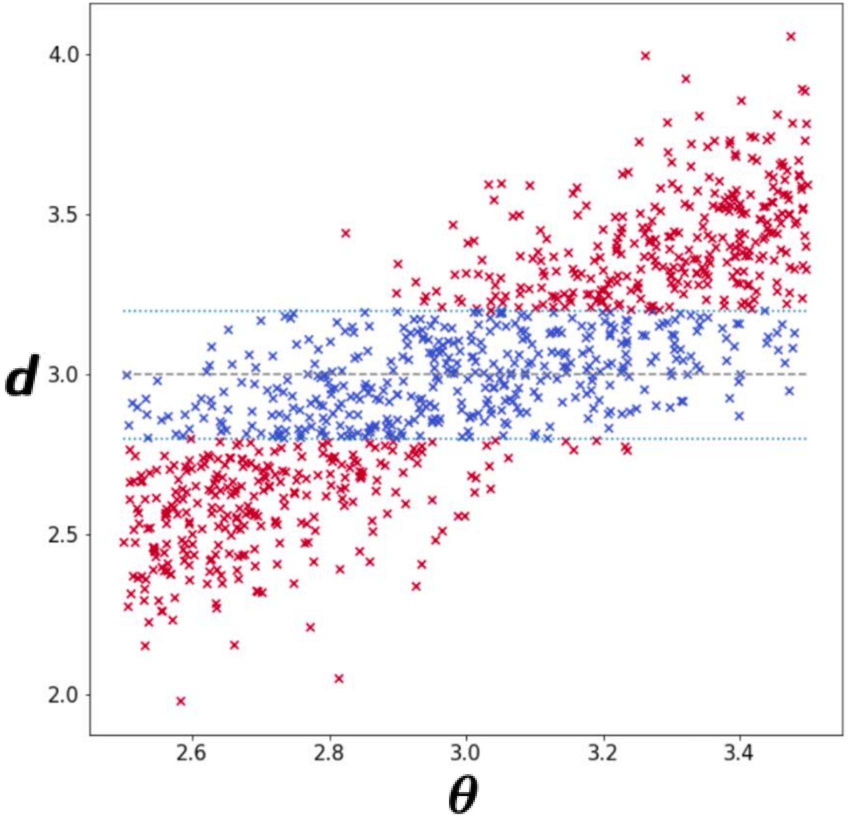
$\boldsymbol{d}$: Data

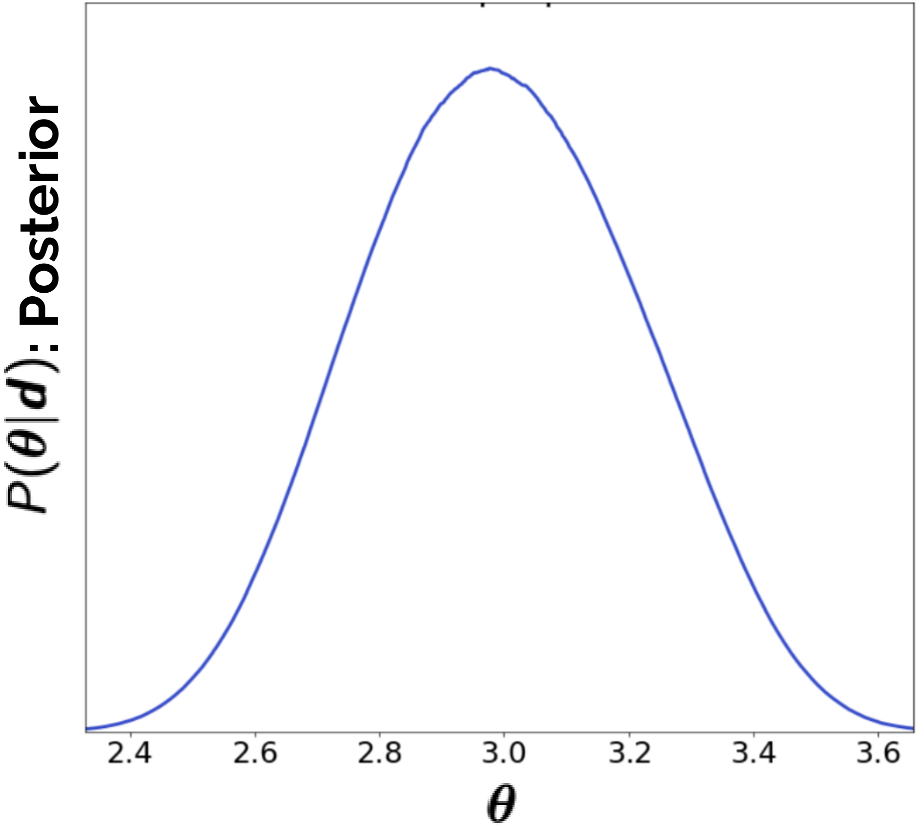
Simulation-based or **likelihood-free** or **implicit likelihood inference**



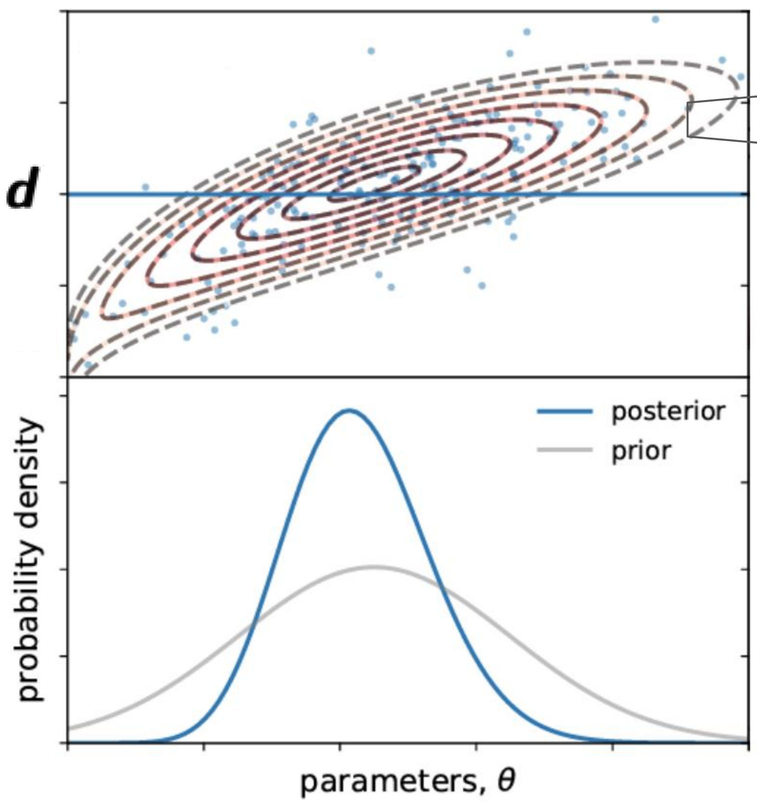
Populate with **simulations** given a model

Approximate Bayesian Computation





Neural Density Estimation



Alsing et al. (2019), MNRAS, 488(3), 4440-4458.

Ensemble of 2 NDEs

2 x MAFs

5 x MADEs

2 hidden layers

50 neurons each

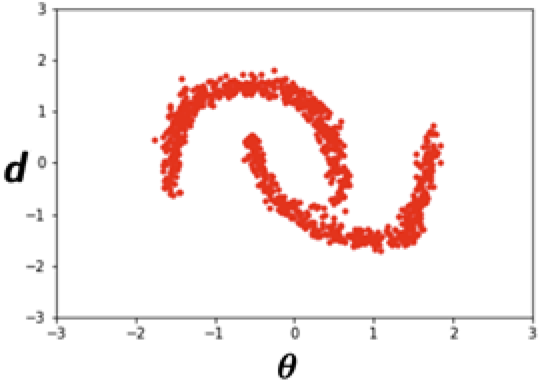
→ 2 x 5 x |***d***| multivariate Gaussians, P(***d***|***θ*****, w**)

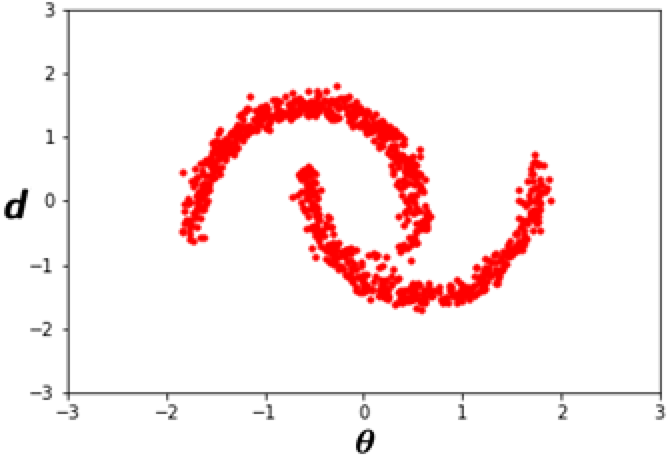
Masked Autoregressive Flows

$P(\theta, d)$
from simulations

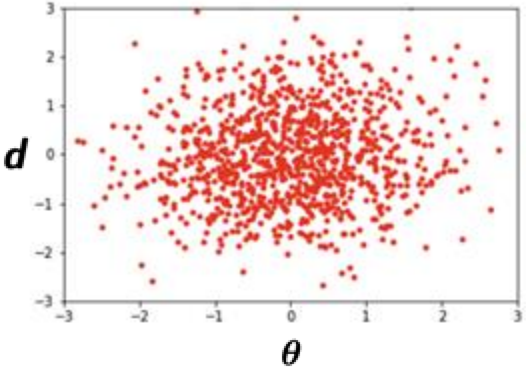
Learn transformations to

Gaussian distribution



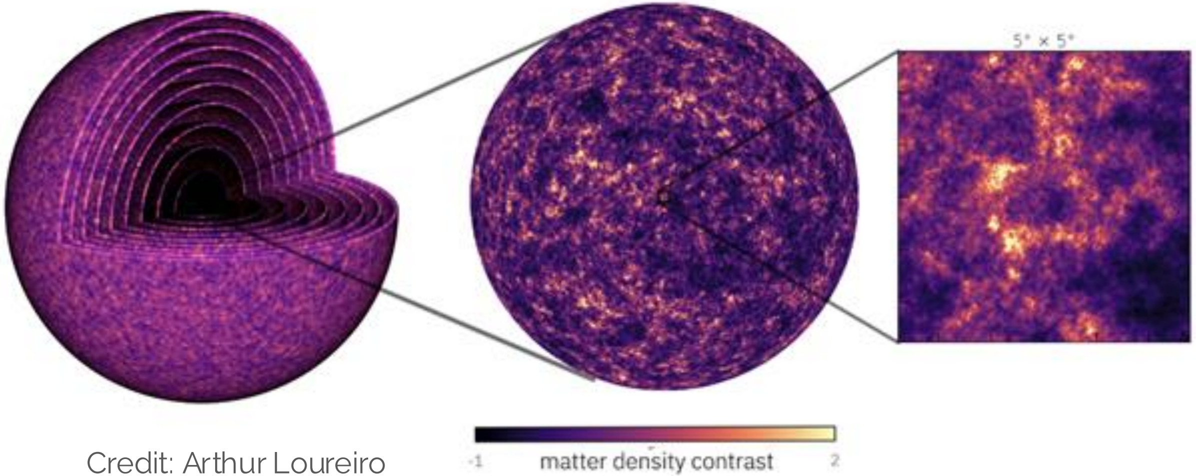


Credit: Eric Jang



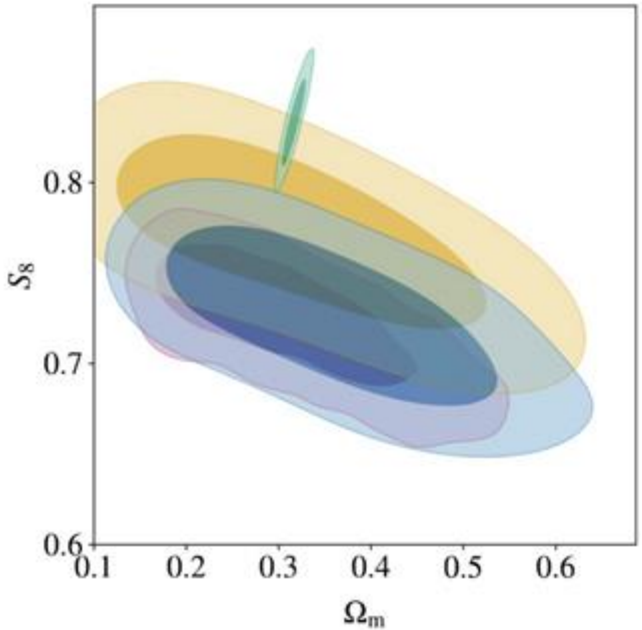
Invertible

e.g. SBI in Cosmic Shear



Credit: Arthur Loureiro

KiDS
Kilo-Degree Survey

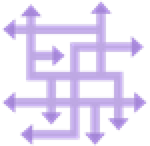


18,000 simulations varying 32 parameters in LambdaCDM + nuisance

von Wietersheim-Kramsta, Lin et al. (2025), A&A 694, A223.

Simulation-Based Inference

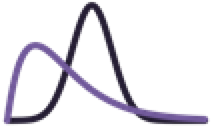
a.k.a. likelihood-free inference or implicit likelihood inference



Signal and uncertainty modelling of arbitrary complexity
(vary all complexities simultaneously)

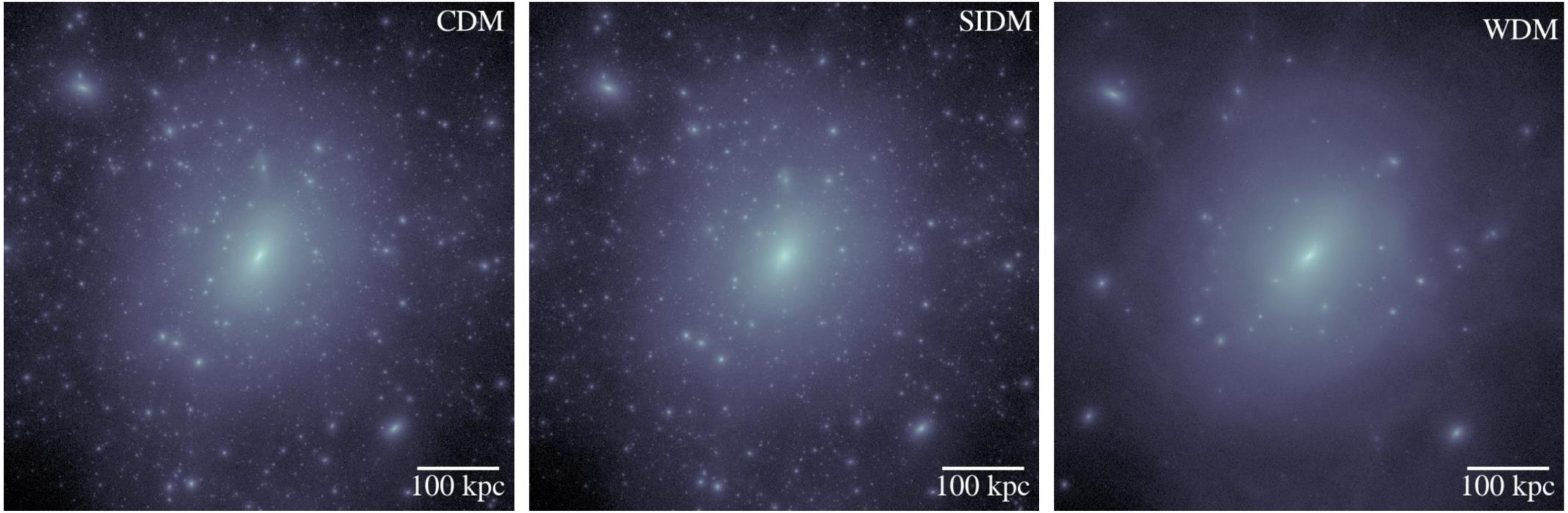
$d_0, d_1, d_2 \ldots$ Can be amortisable (all model evaluations can be data-independent)

$t \rightarrow \Theta$ Bayesian uncertainty propagation from data to parameters



Likelihood can take an arbitrary form

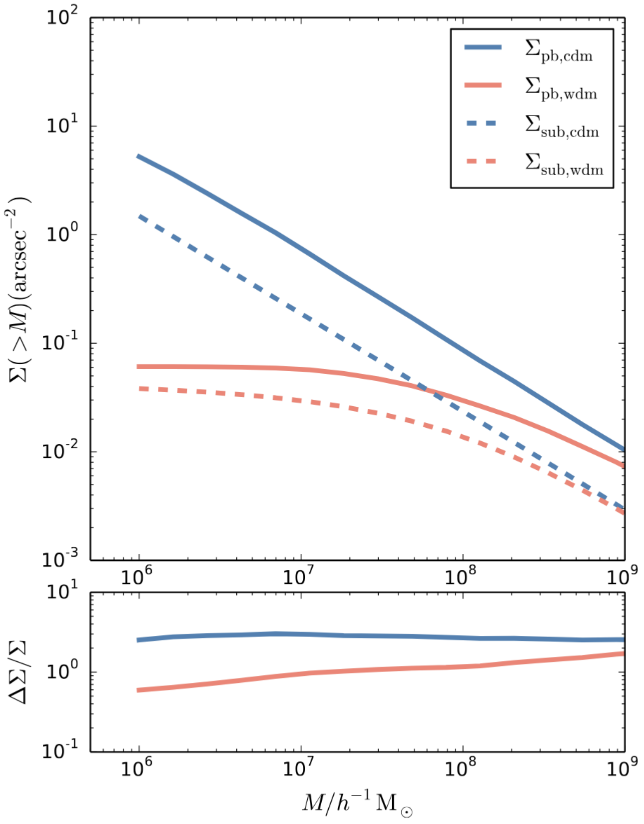
Substructure Search



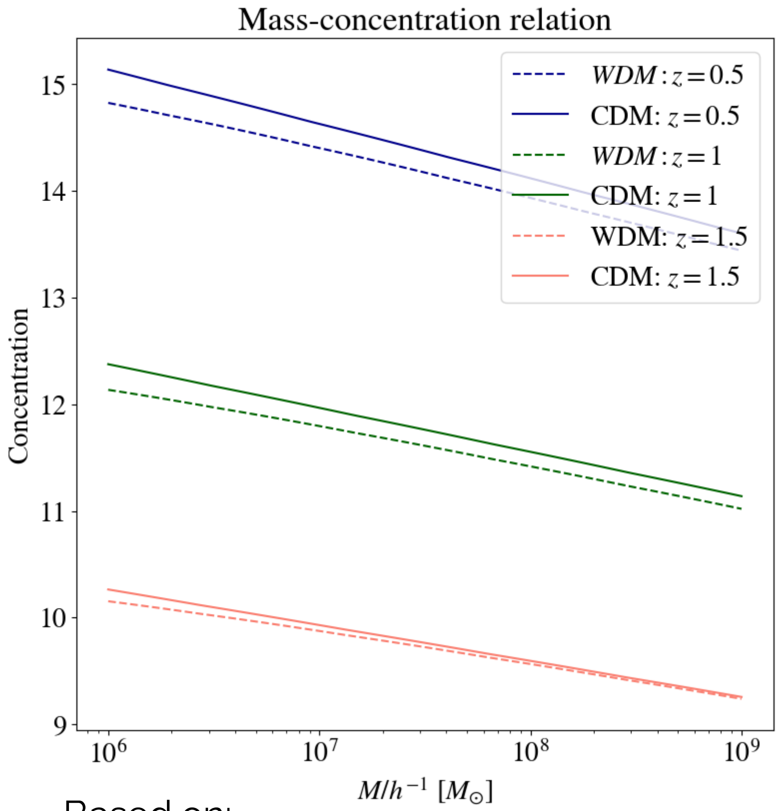
Bullock & Boylan-Kolchin (2017), ARA&A, 55:343-387.

Substructure Search – Forward Modelling

Number densities of **perturbing interlopers and subhaloes**



Li et al. (2016), MNRAS, 468(2), 1426-1432.



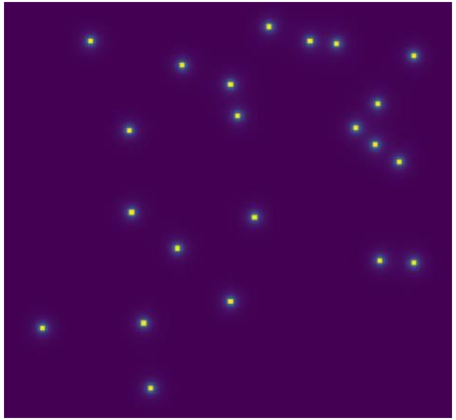
Based on:
Ludlow et al. (2016), MNRAS, 460(2), 1214-1232.

Substructure Search – Forward Modelling

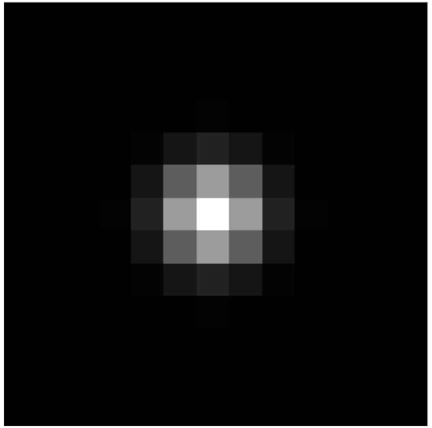
Source:
- Elliptical Core-Sersic
- z = 1

Lens:
- Power law mass
- z = 0.5
- No external shear

Perturbers:
- Warm Dark Matter
- Truncated NFW mass
- $M_{hf} = 10^7$
- $\mathbf{n_{subhalos}} \in \mathbf{[0, 30]}$



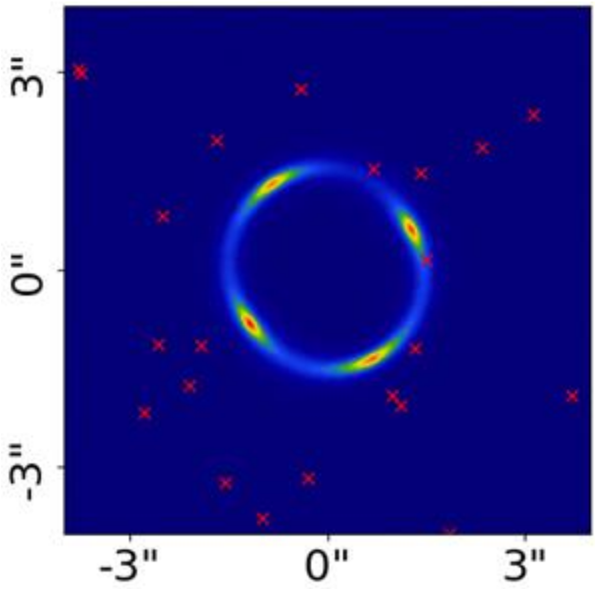
Observational Effects
(HST-like)
- Exposure = 8000s
- Sky background = 0.1
- Pixel scale = 0.05”
- $\boldsymbol{\sigma}_{PSF}$ = 0.05”
- + Poisson noise



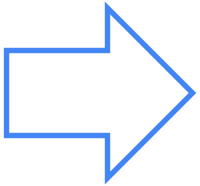
He et al. (2022), MNRAS, 511(2), 3046-3062.

Substructure Search – Forward Modelling

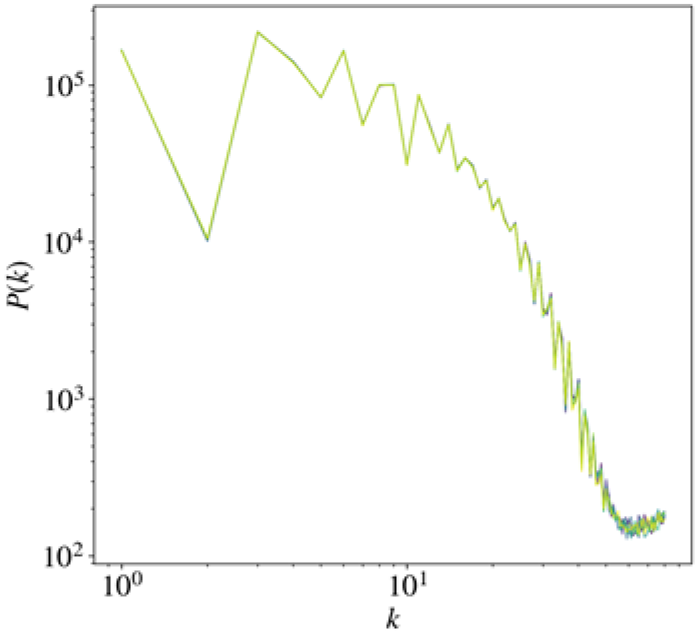
AutoLens: Mock Observation



Nightingale et al., (2021), JOSS, 6(58), 2825

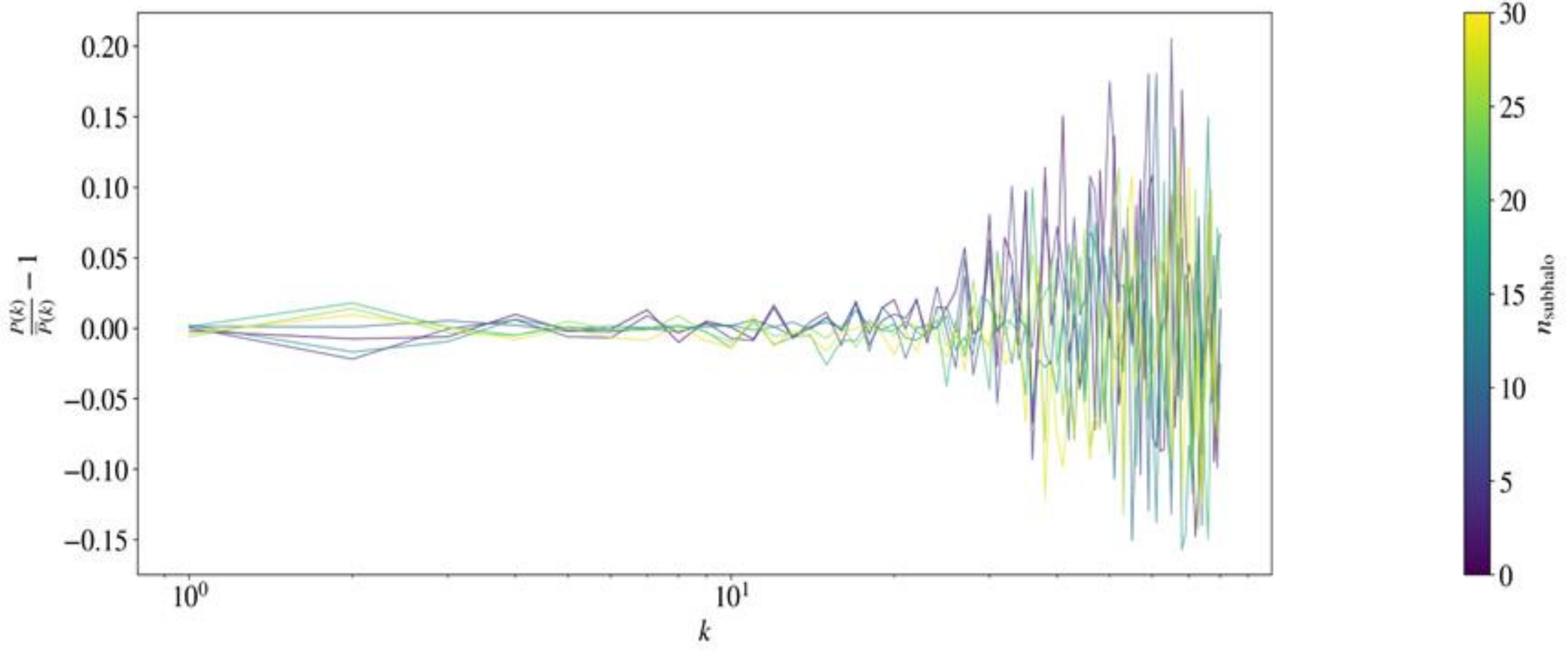


Compression/summary statistic: P(k)



RINSE & REPEAT 1000 TIMES!

Substructure Search – Forward Modelling



Substructure Search – SBI

Ensemble of 2 NDEs

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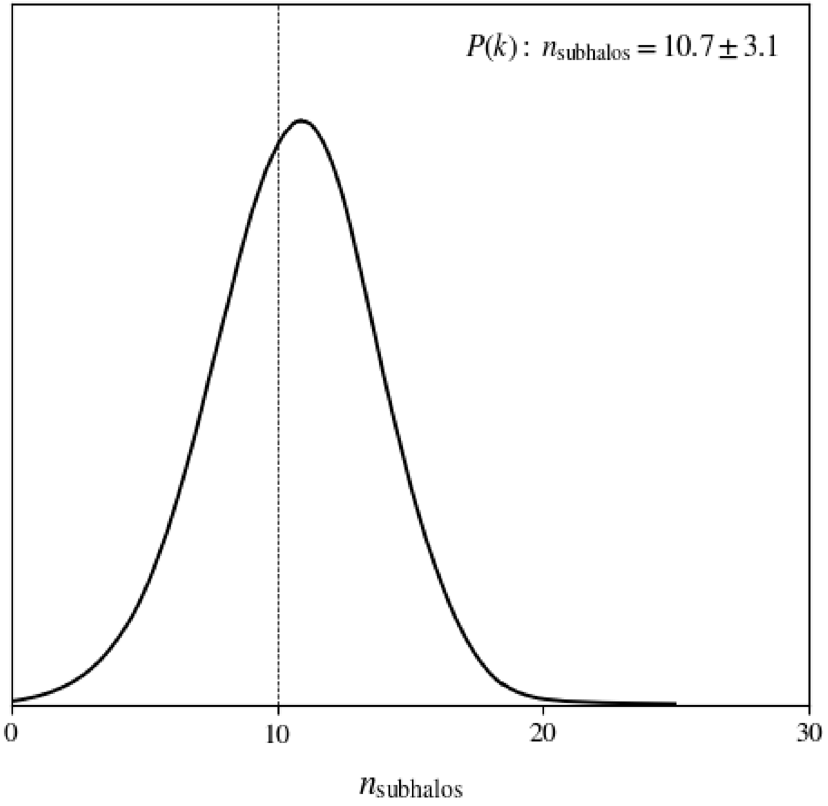
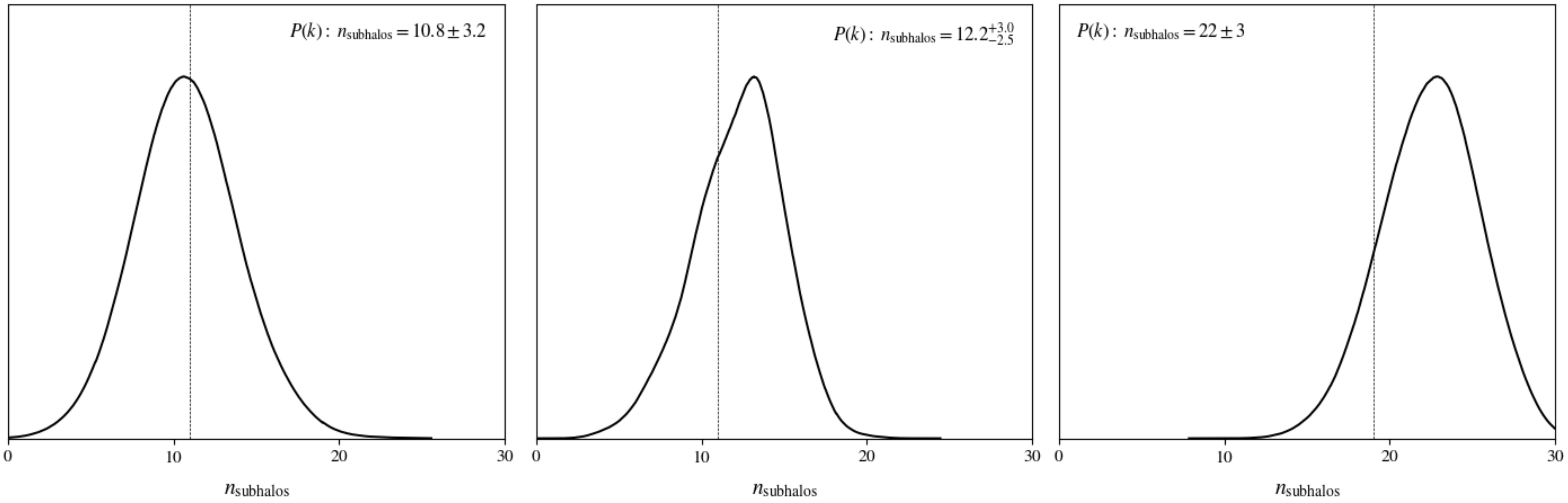
2 hidden layers

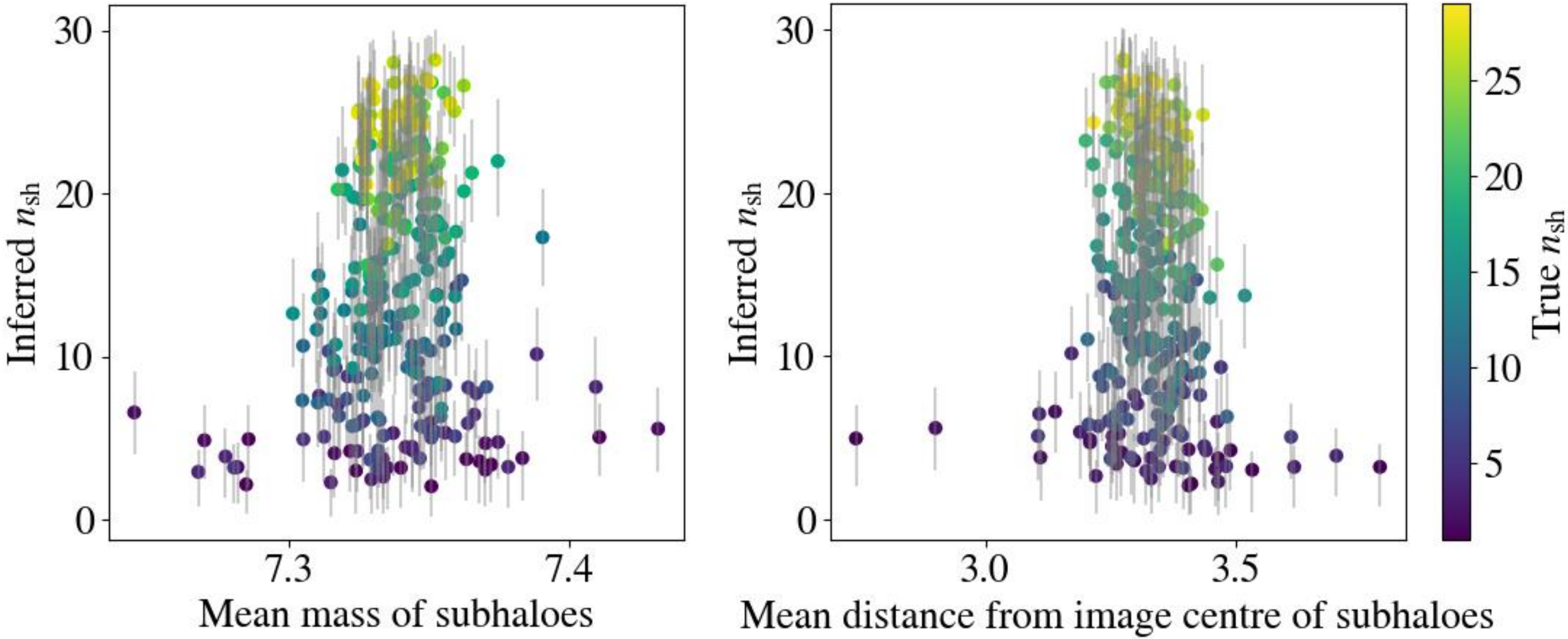
50 neurons each



Substructure Search – SBI



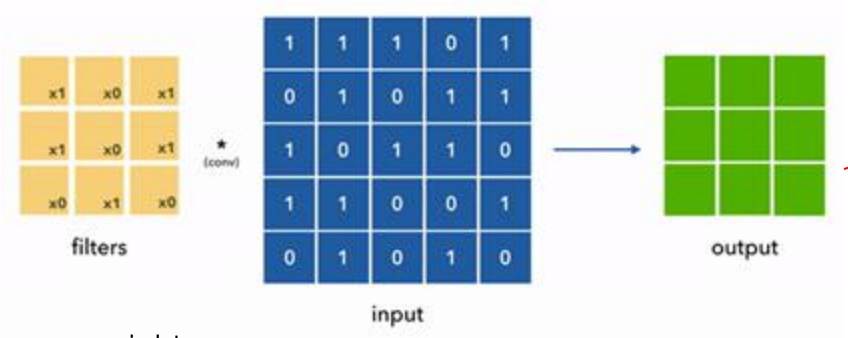
Substructure Search – SBI



Substructure Search – SBI

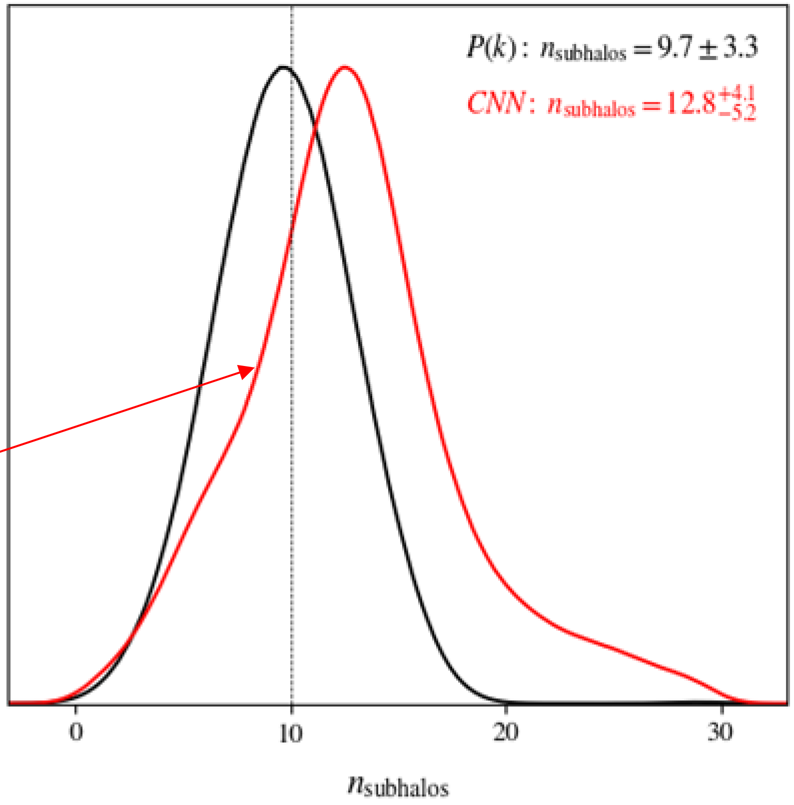
Other compression schemes/summary statistics:

Convolutional Neural Networks



Learn weights based on all simulated images

Convolutional layer



Conclusions & Outlooks

- SBI can incorporate complexities into lens model inference
- We accurately and robustly recover subhalo counts

Plans:

- Scale up to higher-dimensional parameter space
- Add realism and additional dark matter models
- Apply to data